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THE

LYCEUM

ARITHMETIC

IN THREE PARTS.

EACH

ADAPTED TO DIFFERENT AGES AND CL

PREPARED FOR

COMMON SCHOOLS, HIGH SCHOOL
AND ACADEMIES.

BY AN EXPERIENCED TEACHER.

BOSTON:
PUBLISHED BY WILLIAM PEIRCE
1835.

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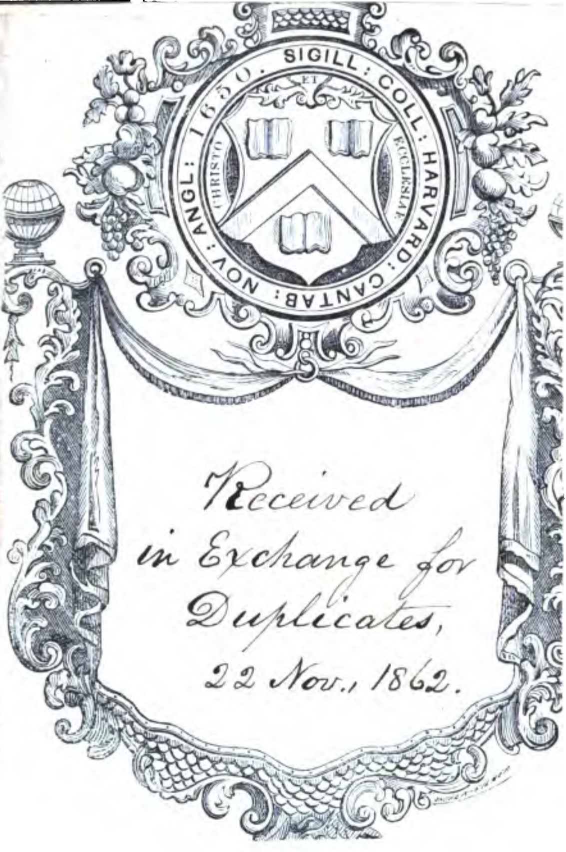

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PREFACE.

The advantages offered in this work are;

First. It is divided into *three courses;* the first for beginners, in which the elements of the whole science are taught in their simplest forms. The second part contains a more extended course. The third part, in connection with the preceding, contains as extended and complete a course of Arithmetic, as any work now used in any school or college. Of course the work is fitted for all kinds of schools, and for pupils of all ages, and all degrees of advancement.

By going over the principles of the whole science, first by simple exercises, and afterwards with the more complicated and difficult, more intelligent and systematic knowledge is gained than is attained by taking each separate subject, in its widest extent, a method which requires nearly the whole work to be completed before some very important principles are reached.

Second. Mental and Written Arithmetic are combined in simultaneous exercises. Some works contain exercises in mental arithmetic alone, to be followed by a course of Written Arithmetic, a plan which most practical teachers have found to involve disadvantages. Other works omit exercises in mental arithmetic altogether, thus leaving out one of the most useful exercises for mental discipline, as well as for practical use, in after life.

Third. In performing mental exercises, this work requires the pupil to do it so clearly, as to be able to state the process aloud; and the manner of doing this is explained and illustrated. Pupils very often arrive at correct results, and yet in so confused a way, as to be unable to trace the process. Requiring them to state aloud this process, is a most valuable aid to mental discipline. In the exercises in written arithmetic also, the *rationale* of every process and rule is explained, so that nothing is to be done mechanically, but every thing intelligently. It is not unfrequently the case that pupils are able to perform all arithmetical written exercises, simply by a mechanical rule, directing that the figures be placed so and so, and then multiplied, divided, &c., while the whole is to them a kind of black art, of which the principles and philosophy are entirely unknown.

Fourth. In teaching Numeration, the nature of Vulgar and Decimal Fractions is taught, and their mode of numeration. This will be found a great advantage when fractions are attended to, on the succeeding pages.

Fifth. This work is intended to aid young and inexperienced teachers, by instructing them in the modes of explaining and illustrating, which have been found most useful in the experience of others

The writer of this work has used in instructing classes, Daboll, Colburn, Adam's, Smith, and other of the most popular arithmetics. Each of these works contains peculiar advantages. The author of this, has endeavored to combine in *one* work, these various excellences, which are scattered among several.

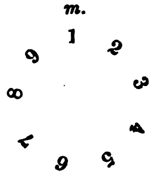
TO TEACHERS.

It is recommended that all pupils, of whatever age or advancement, go over the first part, before taking the second; as, although it seems very simple, it contains important principles, not so clearly explained in any other part.

In teaching Numeration, when young children first commence, it is recommended, that only the easiest exercises in Decimal Numeration be taught, until reviewing.

It is very important that children learn the Addition and Multiplication Tables before proceeding to any other Arithmetical exercises. Many teachers have found the plan of *Circulating Classes* useful in adding interest to this exercise among children. A description is therefore added for any who would like to try the same method.

Let the pupils of the class stand in a circle, according as the figures are placed below.



In saying the Multiplication Table, No. 1 begins and says '3 times 1?' No. 2 answers, 'is 3;' and then turns to No. 3 and says '3 times 2?' No. 3 answers, 'is 6;' and then turns to No. 4 and says, '3 times 3?' No. 4 answers, 'is 9.' Thus it goes round the ring, and continues to circulate around till the lesson is finished.

If any one is *inattentive*, so as not to hear the question, or does not know the answer, the next one answers and goes above the one who neglects to answer. If the next cannot do it, it passes around till some one can answer, and the one who answers correctly, goes to the place above the one who first failed.

At *m* just behind No. 1, the teacher or monitor stands, and in going above those who make mistakes, every time a pupil passes the monitor, a mark of honor is given. For these marks of honor some reward is given at the discretion of the teacher. This method serves to *keep up attention* in the class, and makes the exercise more interesting and useful. The more *regular* and *speedy* the exercise the better.

INDEX.

ARITHMETICAL TABLES.

ARITHMETIC.—PART FIRST.

PART SECOND.

PART THIRD.

ARITHMETICAL TABLES.

ADDITION TABLE.

2	and	0	are	2	4	and	0	are	4	6	and	0	are	6	8	and	0	are	8
2		1		3	4		1		5	6		1		7	8		1		9
2		2		4	4		2		6	6		2		8	8		2		10
2		3		5	4		3		7	6		3		9	8		3		11
2		4		6	4		4		8	6		4		10	8		4		12
2		5		7	4		5		9	6		5		11	8		5		13
2		6		8	4		6		10	6		6		12	8		6		14
2		7		9	4		7		11	6		7		13	8		7		15
2		8		10	4		8		12	6		8		14	8		8		16
2		9		11	4		9		13	6		9		15	8		9		17

3	and	0	are	3	5	and	0	are	5	7	and	0	are	7	9	and	0	are	9
3		1		4	5		1		6	7		1		8	9		1		10
3		2		5	5		2		7	7		2		9	9		2		11
3		3		6	5		3		8	7		3		10	9		3		12
3		4		7	5		4		9	7		4		11	9		4		13
3		5		8	5		5		10	7		5		12	9		5		14
3		6		9	5		6		11	7		6		13	9		6		15
3		7		10	5		7		12	7		7		14	9		7		16
3		8		11	5		8		13	7		8		15	9		8		17
3		9		12	5		9		14	7		9		16	9		9		18

SUBTRACTION TABLE.

1	from	1	leaves	0	4	from	4	leaves	0	7	from	7	leaves	0
1		2		1	4		5		1	7		8		1
1		3		2	4		6		2	7		9		2
1		4		3	4		7		3	7		10		3
1		5		4	4		8		4	7		11		4
1		6		5	4		9		5	7		12		5
1		7		6	4		10		6	7		13		6
1		8		7	4		11		7	7		14		7
1		9		8	4		12		8	7		15		8
1		10		9	4		13		9	7		16		9

2	from	2	leaves	0	5	from	5	leaves	0	8	from	8	leaves	0
2		3		1	5		6		1	8		9		1
2		4		2	5		7		2	8		10		2
2		5		3	5		8		3	8		11		3
2		6		4	5		9		4	8		12		4
2		7		5	5		10		5	8		13		5
2		8		6	5		11		6	8		14		6
2		9		7	5		12		7	8		15		7
2		10		8	5		13		8	8		16		8
2		11		9	5		14		9	8		17		9

3	from	3	leaves	0	6	from	6	leaves	0	9	from	9	leaves	0
3		4		1	6		7		1	9		10		1
3		5		2	6		8		2	9		11		2
3		6		3	6		9		3	9		12		3
3		7		4	6		10		4	9		13		4
3		8		5	6		11		5	9		14		5
3		9		6	6		12		6	9		15		6
3		10		7	6		13		7	9		16		7
3		11		8	6		14		8	9		17		8
3		12		9	6		15		9	9		18		9

MULTIPLICATION TABLE.

2 times 0 are 0	5 times 0 are 0	8 times 0 are 0	11 times 0 are 0
2 × 1 = 2	5 × 1 = 5	8 × 1 = 8	11 × 1 = 11
2 2 4	5 2 10	8 2 16	11 2 22
2 3 6	5 3 15	8 3 24	11 3 33
2 4 8	5 4 20	8 4 32	11 4 44
2 5 10	5 5 25	8 5 40	11 5 55
2 6 12	5 6 30	8 6 48	11 6 66
2 7 14	5 7 35	8 7 56	11 7 77
2 8 16	5 8 40	8 8 64	11 8 88
2 9 18	5 9 45	8 9 72	11 9 99
2 10 20	5 10 50	8 10 80	11 10 110
2 11 22	5 11 55	8 11 88	11 11 121
2 12 24	5 12 60	8 12 96	11 12 132
3 times 0 are 0	6 times 0 are 0	9 times 0 are 0	12 times 0 are 0
3 × 1 = 3	6 × 1 = 6	9 × 1 = 9	12 × 1 = 12
3 2 6	6 2 12	9 2 18	12 2 24
3 3 9	6 3 18	9 3 27	12 3 36
3 4 12	6 4 24	9 4 36	12 4 48
3 5 15	6 5 30	9 5 45	12 5 60
3 6 18	6 6 36	9 6 54	12 6 72
3 7 21	6 7 42	9 7 63	12 7 84
3 8 24	6 8 48	9 8 72	12 8 96
3 9 27	6 9 54	9 9 81	12 9 108
3 10 30	6 10 60	9 10 90	12 10 120
3 11 33	6 11 66	9 11 99	12 11 132
3 12 36	6 12 72	9 12 108	12 12 144
4 times 0 are 0	7 times 0 are 0	10 times 0 are 0	13 times 0 are 0
4 × 1 = 4	7 × 1 = 7	10 × 1 = 10	13 × 1 = 13
4 2 8	7 2 14	10 2 20	13 2 26
4 3 12	7 3 21	10 3 30	13 3 39
4 4 16	7 4 28	10 4 40	13 4 52
4 5 20	7 5 35	10 5 50	13 5 65
4 6 24	7 6 42	10 6 60	13 6 78
4 7 28	7 7 49	10 7 70	13 7 91
4 8 32	7 8 56	10 8 80	13 8 104
4 9 36	7 9 63	10 9 90	13 9 117
4 10 40	7 10 70	10 10 100	13 10 130
4 11 44	7 11 77	10 11 110	13 11 143
4 12 48	7 12 84	10 12 120	13 12 156

1. *Troy Weight.*

24 grains (*gr.*) make	1 penny-weight, marked	*pwt.*
20 penny-weights,	1 ounce,	*oz.*
12 ounces,	1 pound,	*lb.*

2. *Avoirdupois Weight.*

16 drams (*dr.*) make	1 ounce,	*oz.*
16 ounces,	1 pound,	*lb.*
28 pounds, 1 quarter of a hundred weight,		*qr.*
4 quarters,	1 hundred weight,	*cwt.*
20 hundred weight,	1 ton,	*T.*

By this weight are weighed all coarse and drossy goods, grocery wares, and all metals except gold and silver.

3. *Apothecaries Weight.*

20 grains (*gr.*) make	1 scruple,	℈
3 scruples,	1 dram,	ʒ
8 drams,	1 ounce,	℥
12 ounces,	1 pound,	℔

Apothecaries use this weight in compounding their medicines.

4. *Cloth Measure.*

4 nails (*na.*) make	1 quarter of a yard,	*qr.*
4 quarters,	1 yard,	*yd.*
3 quarters,	1 Ell Flemish,	*E. Fl.*
5 quarters,	1 Ell English,	*E. E.*
6 quarters,	1 Ell French,	*E. Fr.*

5. *Dry Measure.*

2 pints (*pt.*) make	1 quart,	*qt.*
8 quarts,	1 peck,	*pk.*
4 pecks,	1 bushel,	*bu.*

This measure is applied to grain, beans, flax-seed, salt, oats, oysters, coal, &c.

6. *Wine Measure.*

4 gills (*gi.*) make	1 pint,	*pt.*
2 pints,	1 quart,	*qt.*
4 quarts,	1 gallon,	*gal.*
31½ gallons,	1 barrel,	*bl.*

42 gallons,	1 tierce,	*tier.*
63 gallons	1 hogshead,	*hhd.*
2 hogsheads,	1 pipe,	*p.*
2 pipes,	1 tun,	*T.*

All brandies, spirits, mead, vinegar, oil, &c. are measured by wine measure. *Note.* — 231 solid inches make a gallon.

7. *Long Measure.*

3 barley corns (*b. c.*) make	1 inch,	marked *in.*
12 inches,	1 foot,	*ft.*
3 feet,	1 yard,	*yd.*
5½ yards,	1 rod, pole or perch,	*rd.*
40 rods,	1 furlong,	*fur.*
8 furlongs,	1 mile,	*m.*
3 miles,	1 league,	*lea.*
69½ statute miles,	1 degree, on the earth.	°

360 degrees, the circumference of the earth.

The use of long measure is to measure the distance of places, or any other thing, where length is considered, without regard to breadth.

N. B. In measuring the height of horses, 4 inches make 1 hand. In measuring depths, six feet make one fathom or French toise. Distances are measured by a chain, four rods long, containing one hundred links.

8. *Land, or Square Measure.*

144 square inches make	1 square foot.
9 square feet,	1 square yard.
30¼ square yards, or 272¼ square feet,	1 square rod.
40 square rods,	1 square rood.
4 square roods,	1 square acre.
640 square acres,	1 square mile.

NOTE.—In measuring land, a chain, called Gunter's chain, 4 rods in length, is used. It is divided into 100 links. Of course, 25 links make a rod, and 25 times 25 = 625 square links make a square rod. In 4 rods, there are 792 inches. Of course, 1 link is $7\frac{92}{100}$.

9. *Solid, or Cubic Measure.*

1728 solid inches make	1 solid foot.

40 feet of round timber, or 50 feet of hewn timber,	1 ton or load.
128 solid feet, or 8 feet long, 4 wide, and 4 high,	1 cord of wood.

All solids, or things that have length, breadth and depth, are measured by this measure. N. B. The wine gallon contains 231 solid or cubic inches, and the beer gallon, 282. A bushel contains 2150.42 solid inches.

10. *Time.*

60 seconds (*S.*) make	1 minute, marked	*m.*
60 minutes,	1 hour,	*h.*
24 hours,	1 day,	*d.*
7 days,	1 week,	*w.*
4 weeks,	1 month,	*mo.*
13 months, 1 day and 6 hours,	1 Julian year,	*yr.*

Thirty days hath September, April, June, and November, February twenty-eight alone, all the rest have thirty-one.

N. B. In bissextile or leap-year, February hath 29 days.

11. *Circular Motion.*

60 seconds (″) make	1 minute,	′
60 minutes,	1 degree,	°
30 degrees,	1 sign,	*S.*

12 signs, or 360 degrees, the whole great circle of the Zodiac.

12. *Sterling Money.*

4 farthings, (*qrs.*) make	1 penny,	marked *d.*
12 pence,	1 shilling,	*s.*
20 shillings,	1 pound,	£

13.

12 units	make	A Dozen.
12 dozen		A Gross.
144 dozen		A Great Gross.
20 units		A Score.
24 sheets of paper		A Quire.
20 quires		A Ream.

Value of Foreign Coins in Federal Money.

Shilling Sterling,	$ 0.222	Rix Dollar of Austria,	0.778—
Crown 5s.	1.111	Rix dollar of Denmark and Switzerland,	1.000
Sovereign, (a gold Coin, = £)	4.444	Rix Dollar of Sweden,	1.037
Guinea, (21s. nearly out of use in England,)	4.666	Rix Dollar* of Prussia,	0.778—
		Florin, "	0.259+
Livre of France,	0.185+	Ducat of Sweden and Prussia,	2.074
Franc "	0.1875—		
Pistole* 10 livres "	1.852—	Piaster of ex, of Spain,	0.80
Louis d'or, "	4.444+	Ducat of ex,* "	1.102—
Five franc piece, "	0.937	Stiver of Holland,	0.019+
Real of Plate, of Spain,	0.100	Guilder or Florin, "	0.388
Real of Vellon, "	0.050	Rix Dollar, "	0.970
Pistole, "	3.60	Ducat, "	2.079—
Dollar, "	1.00	Gold Ducat, "	8.000
Re. of Portugal,	$ 0.0012+	Ducat of Denmark,	8.833+
Testoon, "	0.125	Ruble, of Russia,	1.000
Milre,* "	1.250	Zervonitz. "	2.000
Moidore, "	6.000	Tale, of China,	1.480
Joanese, "	8.000	Pagoda, of India,	1.840
Marc Banco of Hamburgh,	0.333+	Rupee, of Bengal,	0.500
Pistole of Italy,	3.200	Xeriff, of Turkey,	2.222

* Those denominations which have the asterisk, (as the Pistole of France, and the Milre of Portugal,) are merely *nominal*; that is, they are represented by no *real coin*. In this respect, they are like the Mill in Federal Money.

A TABLE OF SCRIPTURE WEIGHTS, MEASURES AND MONEY.

MEASURES OF LENGTH.

	feet.	*inches.*
A Cubit,	1	9.88
A Span, *half cubit*,	0	10.94
A Hand breadth,	0	3.68
A Finger,	0	0.91
A Fathom,	7	3.55
Ezekiel's reed,	10	11.32
The measuring line,	145	11.04

	miles.	*furlongs.*	*rods.*	*feet.*
Sabbath day's journey,	0	5	21	1½
Eastern mile,	1	3	2	3
Stadium, or Furlong,	0	1	4	3
Day's journey,	33	1	12	6

MEASURE OF LIQUIDS.

	gal.	*pints.*	*sol. inch*
The Homer or Cor. - -	75	5	7.6
The Bath, - - -	7	4	15.2
The Hin, - - - -	1	2	2.5
The Log, - - - -	0	0	24.3
The Firkin, - - - -	0	7	4.9

MEASURE OF THINGS.

	bushels.	*pecks.*	*pints.*
The Homer, - - - -	8	0	1.6
The Lethech, - - - -	4	0	0.8
The Ephah, - - - -	0	3	3.4
The Seah, - - - -	0	1	1.1
The Omer, - - - -	0	0	5.1
The Cab, - - - -	0	0	2.9

WEIGHTS.

	lb.	*oz.*	*pwt.*	*gr.*
A Shekel, - - -	0	0	9	2.6
The Maneh, - - -	2	3	6	10.3
A Talent, - - -	113	10	1	10.3

MONEY.

	dolls.	*cents.*	*mills.*
A Shekel, - - - -	0	50	5
The Bekah, (*half Shek.*) - -	0	25	3
The Zuza, - - - - -	0	12	5
The Gerah, - - - - -	0	02	5
Maneh or Mina, - - -	25	29	6
A Talent of Silver, - - -	1,157	85	7
A Shekel of Gold, - - -	8	09	4
A Talent of Gold, - - -	24,285	71	4
Golden Daric or Drachm, - -	4	85	7

	dolls.	*cents.*	*mills*
Piece of silver, (*Drachm*) - -	0	14	3
Tribute money, (*Didrachm*) -	0	28	7
Piece of Silver, (*Stater*) - -	0	57	4
Pound, (*Mina*) - - -	14	35	1
Penny, (*Denarius*) - - -	0	14	3
Farthing, (*Assarium*) - - -	0	00	6
Farthing, (*Quadrands*) - -	0	00	3
Mite, - - - - -	0	00	1

ARITHMETIC.

PART FIRST.

Arithmetic is the science of numbers.

A unit is a whole thing of any kind.

A fraction is a part of a thing.

Thus a dollar is a unit; a man is a unit; a picture is a unit; a bushel of apples is a unit, &c.

A *half* of an apple is a fraction; a *quarter* of a dollar is a fraction; a *third* of a loaf of bread is a fraction, &c. Let the pupil mention other units and fractions.

If an apple is cut into *two* equal parts, each part is called *one half* of the apple. If it is cut into *three* equal parts, each part is called *one third*. If it is divided into *four* equal parts, each part is called *one fourth*. If it is divided into *five* equal parts, each part is called *one fifth*, &c.

If a unit is divided into six equal parts, what is one of those parts called? If a unit is divided into seven equal parts, what is one of those parts called? If a unit is divided into eight equal parts, what is one of those parts called? Into nine? Into twenty? Into an hundred? Into fifteen? Into twenty-two?

How many halves make one unit? How many thirds make one unit? How many fourths? How many fifths? How many sixths? How many sevenths? How many eighths? How many ninths? How many tenths? How many twentieths? How many hundredths?

For illustrating the exercises which immediately follow, the teacher should be provided with a proper number of the several coins of the U. S. viz: eagles, dollars, dimes, cents and mills. As mills have never been coined, round bits of stiff paper may be employed to represent them. The pupil should first see the several coins and learn the value of them.

What is arithmetic? What is a unit? What is a fraction? Mention a unit and a fraction.

Ten mills are one cent.	Ten dimes are one dollar.
Ten cents are one dime.	Ten dollars are one eagle.

How many mills make a cent? What part of a cent is one mill? What part of a cent is two mills? What part of a cent is three mills? What part of a cent is four mills? What part of a cent is five mills? Six mills? Seven mills? Eight mills? Nine mills?

How many cents make a dime? One cent is what part of a dime? Two cents is what part of a dime? Three cents is what part of a dime? Four cents is what part of a dime? Five cents? Six cents? Seven cents? Eight cents? Nine cents?

How many dimes make a dollar? What part of a dollar is one dime? What part of a dollar is two dimes? What part of a dollar is three dimes? Four dimes? Five dimes? Six dimes? Seven dimes? Eight dimes? Nine dimes?

How many dollars make an eagle? One dollar is what part of an eagle? Two dollars is what part of an eagle? Three dollars? Four dollars? What part of an eagle is five dollars? Six dollars? Seven dollars? Eight dollars? Nine dollars?

Order means the same as *kind.*

The *same thing* may be considered sometimes as a unit and sometimes as a fraction—thus, one dollar is a *unit* or whole thing of the *kind* or *order* called dollars, and one dollar is also the *tenth part* of an *eagle*, or the *fraction* of an eagle. One cent is a *unit* or *whole thing*, of the order of cents, and one cent is also the *tenth part* of a *dime*, or the *fraction* of a dime. One mill is a *unit* of the order of mills, and one mill is the *tenth part* of a *cent*, or the *fraction* of a cent. One *day* is a *unit* or whole thing of the order of days, and one day is also the *seventh part* of a *week*, or the *fraction* of a week. One week is a *unit* or *whole thing* of the order of weeks, and one week is the *fourth part* of a *month*, or the *fraction* of a month. One month is a *unit* or whole thing of the order of months, and one month is also the *twelfth part* of a *year*, or the *fraction* of a year.

Of what order is one dollar *a unit?* Of what order is it *a fraction?* Of what order is one cent *a unit?* Of what order is it *a fraction?* Of what order is one week *a unit?* Of what order is it *a fraction?* Of what order is one foot a unit? Of what order is it *a fraction?* One day is *a unit*, of what order, and *a fraction* of what order, &c.?

What is half of four cents? What is a *third* of *six* cents?

What does *order* mean? How may the *same thing* be considered?

Let the pupil take six cents, and divide them into three equal portions, and then tell what is one of these parts?

What is a *fourth* of *eight* cents? Let the pupil divide *eight* cents into *four* equal portions, and tell how many in each portion.

There are *twice* six cents in twelve cents, what part of twelve is six cents?

There are *three* times four cents in twelve cents, what part of twelve is four cents?

There are *three* times five cents in fifteen cents, what part of fifteen is five cents?

There are *three* times three in nine, what part of nine is three?

There are *two* times three in six, what part of six is three?

There are *four* times two in eight, what part of eight is two?

There are *four* times three in twelve, what part of twelve is three?

There are *five* times six in thirty, what part of thirty is six?

There are *three* times seven in twenty-one, what part of twenty-one is seven?

There are *four* times six in twenty-four, what part of twenty-four is six?

There are *six* times seven in forty-two, what part of forty-two is seven?

What part of twelve is three? Is four?

What part of nine is three?

What part of fifteen is three? Is five?

What part of sixteen is four?

What part of eighteen is three? Is six?

What part of twenty-one is three? Is seven?

What part of twenty-four is six? Is four?

What part of twenty-eight is seven? Is four?

What part of thirty-two is eight? Is four?

What part of thirty-six is nine? Is four?

If an apple is cut into two equal parts, what is each part called? If it is cut into three equal parts, what is each part called?

The more parts a thing is divided into, the *smaller* these parts must be. If one thing is divided into twice as many parts as another thing, each part is twice as small.

If one apple is cut into twice as many pieces as another, how much smaller is each piece? How much larger is a half than a fourth? Ans. There are twice as many fourths as halves in a thing, therefore a half is twice as large as a fourth.

If one apple is cut into four pieces, and another into eight pieces, how much larger are the *fourths* than the *eighths?* Ans. As there are *twice as many* pieces when there are eighths, as when there are fourths, an eighth is *twice as small* as a fourth.

If one apple is cut into twelve parts and another into six parts, which has the *most* parts and which has the *largest* parts? How much larger is a sixth than a twelfth? Ans. Twelve is *twice as many* as six, therefore a sixth is twice as *large* as a twelfth.

Which is the largest, a fifth or a tenth? How much larger is a fifth than a tenth?

Which is the largest a seventh or a fourteenth?

How much smaller is a fourteenth than a seventh?

Which is the largest a third or a fifth?

Which is the smallest a half or a fourth?

Which is the smallest a third or a half? Ans. The *more* pieces there are, the *smaller* they must be, therefore a third must be smaller than a half.

If one apple was cut into four pieces, and another into six pieces, which would be the largest a fourth or a sixth?

Which is the largest a sixth or a ninth?

Which is the largest a fifth or a fourth?

Which is the smallest a twelfth or a tenth?

Which is the smallest a seventh or a ninth?

Which is the smallest an eighth or a seventh?

Which is the smallest a fifteenth or a fifth?

Which is the largest an eighth or a sixteenth?

Which is the largest a fifth or a half?

If an apple is divided into four pieces, what is each piece? If it is divided into twice as many and twice as small pieces, how many are there, and what are they called?

If an apple is divided into thirds, what would you change them to, to make them twice as many and twice as small?

Make two fourths twice as small and twice as many pieces, and what is the answer?

What part of a thing is twice as small as a half? As a third? As a fourth? As a fifth? As a sixth? As a seventh? As an eighth? As a ninth? As a tenth? As an eleventh? As a twelfth?

What part of a thing is twice as large as a fourth? As a sixth? As an eighth? As a tenth? As a twelfth? As a fourteenth? As a sixteenth? As an eighteenth? As a twentieth?

ADDITION.

Two cents, and four cents, and six cents, and nine cents, are how many? Sixteen cents, and twelve cents, are how many?

Five dollars, and four dollars, and nine dollars, are how many?

Four halves of an apple, and six halves, and nine halves, are how many halves?

Five sixths of an apple, and four sixths, and nine sixths, are how many sixths?

Three fifths of an orange, and four fifths, and nine fifths, and twelve fifths, are how many fifths?

Addition is uniting several numbers in one.

When *whole numbers* are added, it is Simple Addition. When *fractions* are added, it is Fractional Addition.

Six dimes, five dimes, and four dimes, are how many?

Seven dollars, eight dollars, and nine dollars, are how many?

Nine cents, three cents, twelve cents, and ten cents are how many?

Four, three, and seven are how many?

Eight, five and three are how many?

Nine, six and two, are how many?

Seven, five, and six are how many?

Eight, nine, and two are how many?

Seven, eight, and one are how many?

Eleven, five, and six are how many?

Ten, seven, and three are how many?

Ten twentieths, six twentieths, and five twentieths are how many twentieths?

One thirteenth of a unit, four thirteenths, and seven thirteenths are how many thirteenths?

One fifth of a dollar, three fifths, and eight fifths are how many fifths?

One ninth of an orange, four ninths, and six ninths are how many ninths?

Seven tenths of an eagle, two tenths, and five tenths are how many tenths?

Three eighteenths, nine eighteenths, and four eighteenths are how many eighteenths?

Ten thirtieths, six thirtieths, and five thirtieths are how many thirtieths?

What is addition? What is simple addition? What is fractional addition?

Two fourths, six fourths, nine fourths, ten fourths, and five fourths, are how many fourths?

Sixteen halves, five halves, nine halves, and six halves, are how many halves?

Six eighths, four eighths, seven eighths, sixteen eighths, are how many eighths?

The number made by adding several numbers together, is called the sum.

What is the sum of four, six, nine and five?

What is the sum of four tenths, six tenths, and nine tenths?

SUBTRACTION.

If you take two cents from three cents, how many remain?

If you take three dollars from six dollars, how many remain?

If you take four dollars from seven dollars, how many remain?

If you take five eagles from nine eagles, how many remain?

If you take six dimes from ten dimes, how many remain?

If two tenths are taken from four tenths, how many remain?

If four ninths are taken from eight ninths, how many remain?

If two tenths are taken from seven tenths, how many remain?

Subtraction is taking one number from another, to find the remainder.

When whole numbers are subtracted it is Simple Subtraction. When fractions are subtracted, it is Fractional Subtraction.

What is the remainder, when four cents are taken from nine cents?

What is the remainder, when three mills are taken from eight mills?

What is the remainder, when seven dimes are taken from twelve dimes?

What is the remainder, when five dollars are taken from ten dollars?

Five from eleven? Seven from thirteen? Eight from twelve? Five from fourteen? Nine from sixteen? Five from twelve? Eight from thirteen? Ten from twenty?

What is the remainder, when two sevenths of an apple are taken from eight sevenths? When four sevenths of a dollar are taken from six sevenths? Eight twelfths from ten twelfths?

What is the number made by adding several numbers together called? What is subtraction? What is simple subtraction? What is fractional traction?

Three ninths from eight ninths? Ten twentieths from twelve twentieths? Six elevenths from ten elevenths? Seven twelfths from twelve twelfths? Eight ninths from thirteen ninths? Three sevenths from nine sevenths? Four eighths from eleven eighths? Four thirds from twelve thirds? Five twentieths from seven twentieths?

The number which has a number subtracted from it, is called the minuend.

The number which is to be subtracted from another number is called the subtrahend.

If eight is subtracted from twelve, what is the subtrahend and what is the minuend?

If four tenths is subtracted from nine tenths, what is the subtrahend and what the minuend?

If ten cents be taken from thirteen cents, what is the subtrahend, and what the minuend?

MULTIPLICATION.

If you take two cents, *three times*, what is the amount of the whole?

If you take three dollars, *four times*, what is the amount of the whole?

If you take half of an apple, *three times*, what is the amount?

If you take two thirds of a dollar, *four times*, what is the amount?

If you take two fourths of an eagle, *six times*, what is the amount?

Multiplication is repeating a number as often as there are units in another number.

If you take five dollars *four times*, what is the amount?

If you repeat four dollars *five times*, what is the amount?

If you take six dollars *five times*, what is the amount?

If you repeat six dollars *six times*, what is the amount? Seven times? Eight times?

If you take seven dollars *three times*, what is the amount?

If you repeat seven *four times*, what is the amount? Five times? Six times? Seven times?

If you repeat eight *twice*, what is the amount?

If you repeat eight *three times*, what is the amount? Four

What is the minuend, and what the subtrahend? What is multiplication?

times? Five times? Six times? Seven times? Eight times?

If you repeat nine *three times*, what is the amount? &c.

If you take one fifth of a dollar *six times*, what is the amount? Seven times? Eight times? Nine times?

If you repeat two sixths of a dollar *three times*, what is the amount?

If you repeat two sixths of a thing *four times*, what is the amount? Five times? Six times? Seven times? Eight times?

What is the amount, if four sevenths be repeated four times? Five times? Six times? Seven times? Eight times?

What is the amount if *five ninths* be repeated *eight times?* Nine times? Ten times? Eleven times?

What is the amount, if eight twentieths be repeated seven times? Nine times? Eight times? &c.

The number to be repeated, is the multiplicand.

The number which shows how often the multiplicand is to be repeated, is called the multiplier.

The multiplier and multiplicand together, are called the factors.

The answer obtained is called the product.

If *eight* is repeated *four* times what is the product? What is the multiplier? The multiplicand? The factors?

If *three sixths* are repeated *four* times what are the factors? The multiplier? The multiplicand?

If you take a *fourth of twelve* and repeat it *three* times, what is the multiplicand? The multiplier? The product?

If you take a *sixth of eighteen* and repeat it *three times*, what is the product? Factors? Multiplier? Multiplicand?

Simple Multiplication is where both factors are whole numbers.

Fractional Multiplication is where one or both factors are fractions.

If twelve is repeated four times, is it simple or fractional multiplication?

If *one fourth of twelve* is repeated three times, is it simple or fractional multiplication? If one sixth is repeated seven times, which kind of multiplication is it?

Exercises in Simple Multiplication.

1. If a man spends three dollars a week, how much does he spend a month?

What is the multiplicand and what the multiplier? What are the mul[illegible]nd and multiplier together called? What is the answer called?

Let the pupil *state* the sum in this manner.

As there are four weeks in a month, a man will spend four times as much in a month, as in a week; four times three is twelve. He will spend twelve dollars.

Let all the following sums be stated in the same way. Both teachers and pupils will find great advantage in being particular to follow this method of stating.

2. If a man spend five dollars a month, how much does he spend in a year?

3. If a man can make eight pens in a minute, how many can he make in ten minutes?

4. If one orange cost six cents, what cost eight oranges?

5. Eight boys have seven cents apiece, how much have all?

6. There is an orchard in which there are six rows of trees, and seven in each row, how many trees in the orchard?

7. The chess board has eight rows of blocks, and eight blocks in each row, how many blocks in the whole?

8. Twelve young ladies have each five books apiece, how many have they all?

9. If a young lady spends six cents a week, how much does she spend in a month?

10. There are nine desks in a school room, and six scholars at each of the desks, how many are in the room?

11. There are in a window five rows of panes of glass, and seven panes in each row, how many in the whole?

12. If one lemon cost four cents, how much will twelve lemons cost?

EXERCISES IN FRACTIONAL MULTIPLICATION.

Multiplication of Fractions by Whole Numbers.

1. If you repeat *one half four* times, what is the product?

2. If you multiply *three fourths* by seven, what is the product?

3. What is *two thirds* multiplied by eight?

4. If a man spend two twelfths of a dollar a day, how many twelfths does he spend in a week?

Ans. As there are seven days in a week, a man spends seven times as much in a week as in one day. Seven times two twelfths is fourteen twelfths. He spends fourteen twelfths of a dollar in a week.

What is simple multiplication? What is fractional multiplication? What is the method of stating?

5. If a man gives two eighths of a pound of meat to six persons, how many eighths does he give away?

6. If a boy gives *two fourths* of an orange to seven of his companions, how many fourths does he give away?

7. If a man drinks *three fourths* of a pint of brandy a day, how many fourths does he drink in a week?

8. What is three times *three eighths?* Six times *six sevenths?*

9. If a man lays by *two eighths* of a dollar a day, how much does he save in a week?

10. If there are two thirds of a pound of meat for each one in a family of seven, how much is there in the whole?

11. What is six times four tenths?

12. What is nine times two thirds?

13. What is seven times four ninths?

14. What is eight times six tenths?

15. What is twelve times two fourths?

16. What is nine times three tenths?

17. What is five times three sixteenths?

18. What is six times seven twentieths?

The *multiplication of whole numbers by fractions*, is deferred to the *Second Part*, because it involves the process of *Division*, which must first be explained.

DIVISION.

How many two cents are there in four cents?

How many two cents in six cents?

How many two cents in eight?

How many two cents in ten?

How many two cents in twelve?

How many three cents are there in six cents? How many in nine? How many in twelve?

How many four cents are there in eight? How many in twelve?

How many five cents are there in ten?

What part of two cents is one cent?

What part of four cents is two? What part of six is two? What part of eight is two? What part of ten is two? What part of twelve is two?

Three cents is what part of six? Three is what part of nine? twelve?

What part of eight is four? What part of twelve is four?

What part of five cents is one? What part of five is two? What part of five is three? Four? Five? Six? &c.

How many two sixths are there in four sixths?

How many three fourths are there in six fourths?

How many four twelfths in eight twelfths?

What part of two twelfths is one twelfth?

What part of four twelfths is two twelfths?

What part of nine twelfths is three twelfths?

Division is finding how often one number is contained in another, and thus finding what part of one number is another number.

How many times is six contained in twelve? In eighteen? What part of eighteen is six? What part of twelve is six?

How many times is five contained in ten? In fifteen?

Five is what part of ten? Of fifteen?

How many times is seven contained in fourteen? In twenty-one?

What part of fourteen is seven? What part of twenty-one is seven?

How many times is nine contained in eighteen?

How many times is ten contained in twenty? In thirty? In forty?

What part of sixteen is four?

What part of eighteen is six?

What part of sixteen is eight?

One is what part of thirty? Two is what part of thirty? Three is what part of thirty? Six? Eight? Eleven? Fourteen? Twenty is what part of thirty? &c.

How many two sevenths are there in ten sevenths?

How many three eighths are there in nine eighths?

How many six tenths in eighteen tenths?

How many seven ninths in twenty-one ninths?

How many five elevenths in twenty elevenths?

How many three eighteenths are there in twelve eighteenth?

Two sixths is what part of four sixths?

Two sevenths is what part of ten sevenths?

Three eighths is what part of nine eighths?

What part of eighteen tenths is six tenths?

What part of fourteen ninths is seven ninths?

What part of fifteen elevenths is five elevenths?

What is division?

What part of twelve eighteenths is three eighteenths?

The number which is divided is called the Dividend.

The number by which you divide is called the Divisor.

The answer is called the Quotient.

If you find how many times *three* there are in *twelve*, which is the Divisor? The Dividend? The Quotient?

If *twelve* is divided by *six*, which is the Dividend? The Divisor? The Quotient?

When whole numbers are divided by whole numbers, it is called *Simple Division.*

When either the divisor or dividend is a fraction, it is called *Fractional Division.*

Exercises in Simple Division.

1. If you divide 12 cents equally among three boys, how many will each one have?

Ans. Each one will have as many as there are *threes* in *twelve;* or *four cents.*

2. If there are forty-eight panes of glass in a window, and there are eight panes in each row, how many rows are there?

Ans. As many as there are eights in forty-eight; or *six rows.*

3. How much broadcloth, at six dollars a yard, can you buy for twenty-four dollars?

4. How many hours would it take you to travel twenty-one miles, if you travelled three miles an hour?

5. If you divided thirty-six apples equally among four boys, how many would you give them apiece?

6. How many pounds of raisins, at nine cents a pound, can you buy for sixty-three cents?

7. How many reams of paper, at seven dollars a ream, can you buy for forty-nine dollars?

8. A man agreed to work eight months, for seventy-two dollars, how much did he receive a month?

9. If you buy a bushel of pears for forty-eight cents, how much are they a peck?

10. If there are six shillings in a dollar, how many dollars in thirty-six shillings?

11. Four men bought a horse for forty-eight dollars, what did each man pay?

What is the dividend and what the divisor? What is the answer call- What is simple division? What is fractional division?

12 A man gave sixty-three cents for a horse to ride nine miles, how much was that for each mile?

13. A man agreed to pay eight cents a mile for a horse, and he paid sixty-four cents, how many miles did he go?

14. A man had forty-two dollars, which he paid for wood, at seven dollars a cord, how many cords did he buy?

15. Two boys are running, and are forty-eight rods apart. The hindermost boy gains upon the other, three rods a minute, in how many minutes will he overtake the foremost boy?

16. A vessel contains sixty-three gallons, and discharges seven gallons an hour, in how many hours will it be emptied?

17. If you wish to put sixty-four pounds of butter in eight boxes, how many pounds would you put in each box?

EXERCISES IN FRACTIONAL DIVISION.

Division of Whole Numbers by Fractions.

1. How many halves are there in six oranges?

2. How many thirds are there in four apples?

Ans. *One* apple has *three* thirds, *four* apples have *four times* as many, or *twelve thirds.*

3. How many fourths are there in three oranges?

4. How many fifths are there in four apples?

5. How many sixths are there in two oranges?

6. How many half dollars are there in four dollars?

7. How many quarters of a dollar in five dollars?

8. How many half eagles in eight eagles?

9. In two dollars how many *thirds* of a dollar?

10. If there are six *one* thirds in two dollars, How many *two* thirds are there?

Ans. There are only half as many *two* thirds as there are *one* thirds, or *three* two thirds.

11. In two dollars, how many one sixths? How many two sixths?

12. A man divided two dollars among his workmen, and gave them a third of a dollar apiece, how many workmen had he?

13. A man divided four dollars equally among his children, and gave them each *two thirds* of a dollar, how many children had he?

Ans. As many children as there are *two thirds* in four dollars. In four dollars there are twelve *one* thirds. There are *half* as many *two* thirds, or *six*. He had *six* children.

14. If a man gave two sevenths of a dollar to each of his servants, and gave away in the whole four dollars, how many servants had he?

15. How many two sixths in four?

16. How many two eighths in four?

17. How many two thirds in eight?

18. How many two ninths in six?

19. How many two twelfths in two?

20. How many two twelfths in four?

The division of Fractions by whole numbers is omitted till the Second Part.

REDUCTION.

One dime is how many cents? How many mills?

One unit of the order of dollars, is how many units of the order of dimes? How many of the order of cents? How many of the order of mills?

One eagle is how many dollars? How many dimes? Cents?

One unit of the order of dimes is how many units of the order of cents?

Reduction is changing units of one order, to those of another.

A unit of the order of eagles is how many units of the order of dollars? Of dimes?

Two eagles are how many dollars? How many dimes?

How many dollars in two hundred cents?

How many dollars in twenty dimes?

Thirty units of the order of dimes, are how many units of the order of dollars?

Two pints are one quart.

Eight quarts are one peck.

Four pecks are one bushel.

Two units of the order of quarts, are how many units of the order of pints?

Eight pints are how many quarts?

Two bushels how many pecks?

Eight pecks how many bushels?

Three barley-corns are one inch.

Twelve inches are one foot.

What is reduction?

Three feet are one yard.

One inch is how many barley-corns? Two inches are how many?

Twelve barley-corns are how many inches?

One foot is how many inches? Three feet how many?

One yard is how many feet? How many inches? How many barley-corns?

Two yards are how many feet? How many inches? How many barley-corns?

Three yards are how many feet? How many inches? How many barley-corns?

How many feet are there in five yards? How many inches in five yards? How many barley-corns?

How many barley-corns are there in seven yards?

From the preceding exercises, you learn that *a unit of one order may contain several units of another order.*

What do you learn from the preceding exercises?

How many units of the order of cents, are there in one unit of the order of dimes?

How many units of the order of dollars, are there in one unit of the order of eagles?

How many units of the order of mills, are there in one unit of the order of cents?

How many units of the order of pints, are there in one unit of the order of quarts?

How many units of the order of pecks, are there in one unit of the order of bushels?

How many units of the order of barley-corns, are there in one unit of the order of inches?

How many units of the order of feet, are there in one unit of the order of yards?

How many units of the order of days, are there in one unit of the order of weeks?

How many units of the order of weeks, in one unit of the order of months?

Change two units of the order of dimes, to units of the order of cents.

Change twenty units of the order of cents, to units of the order of dimes.

Change three units of the order of yards, to units of the order of feet.

Change nine units of the order of feet, to units of the order of yards.

Change ten units of the order of pints, to units of the order of quarts.

Change five units of the order of quarts, to units of the order of pints.

Change twenty-one units of the order of days, to units of the order of weeks, &c.

When units of one order are changed to units of a *higher* order, the process is called *Reduction ascending;* and when units of one order are changed to those of a *lower* order, the process is called *Reduction descending.*

If twenty cents are changed to dimes, which kind of reduction is used?

If twenty cents are changed to mills, which kind of reduction is used?

If four gallons are changed to pints, which reduction is used?

If eight feet are changed to inches, which kind of reduction is used?

In changing twelve barley-corns to inches, which kind of reduction is used?

In changing fourteen days to weeks, which reduction is used?

In changing five hours to minutes, which reduction is used?

In changing one hundred and twenty minutes to hours, which reduction is used?

Reduce three dimes to cents; to mills. Which kind of reduction is it?

Reduce three hundred mills to cents; to dimes; and which kind of reduction is it?

Reduce three hundred mills to dollars, and which kind of reduction is it?

Reduce two halves to quarters, and which kind of reduction is it?

Ans. As a half is of more value, it is a higher order than a quarter, therefore it is reduction descending.

In performing this last exercise, the pupil will find the necessity for the following distinction in regard to *units.*

A unit has been defined as "any *whole* thing of a kind," and a fraction is defined as "a part of a thing."

But it is very often the case, that *fractions* are *considered* as

What is reduction ascending? What is reduction descending?

units. Thus when we reduce *quarters* to *halves*, and *halves* to *quarters*, we change units of the order called *quarter*, to units of the order called *half*.

When we say a *whole quarter* of an apple, and a *half a quarter* of an apple, we think of a quarter as a *whole thing of its kind*.

The difference between the two kinds of units is this: when we think of a *whole quarter*, we think of another thing of which the quarter is a part. We think of it as a whole thing in one respect, and as a part of a thing in another respect. But when we think of a *whole apple*, we do not *necessarily* think of another thing of which it is a part.

When we think of a *half* of a loaf of bread, do we think of something of which the half is a part?

When we think of a biscuit, do we necessarily think of something of which it is a part?

When we think of a third of an orange, do we necessarily think of something of which it is a part?

When we think of a house, do we necessarily think of any thing of which it is a part?

Those units which do not require us to think of any other thing of which they are parts, are called *whole numbers*, and those units which do require us to think of other things of which they are parts, are called *fractions*.

What is the difference between units that are whole numbers, and units that are fractions?

Reduce two yards to quarters, and which kind of reduction is it?

Reduce twenty-four inches to feet, and which kind of reduction is it?

Reduce three feet to inches, and which kind of reduction is it?

Which is of highest value, a half or a quarter?

Reduce eight quarters to halves, and which kind of reduction is it?

Reduce two halves to quarters, and which kind of reduction is it?

Reduce sixteen quarters to halves, and which kind of reduction is it?

Reduce two fifths to tenths; six tenths to fifths; eight tenths

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Are *fractions* ever considered as *units*? Give an example. What is the difference between these two kinds of units? Which kind of units are called *whole numbers*, and which are called *fractions*?

to fifths; twelve tenths to fifths; three fifths to tenths; six fifths to tenths.

Reduce one seventh to fourteenths; four fourteenths to sevenths; four sevenths to fourteenths; eight fourteenths to sevenths.

Reduce two sixths to twelfths; four twelfths to sixths; eight twelfths to sixths; five sixths to twelfths.

SUMMARY OF DEFINITIONS.

A *unit* is any whole thing of a kind.

A *fraction* is a part of a thing.

Addition is uniting several numbers in one.

Subtraction is taking one number from another, to find the remainder.

The largest number is the *minuend*, the smallest number is the *subtrahend*.

Multiplication is repeating one number as often as there are units in another number.

The *multiplicand* is the number to be repeated; the *multiplier* is the number which shows *how often* the multiplicand is to be repeated; the *factors* are both the multiplier and multiplicand; and the *product* is the number obtained by multiplying.

Division is finding how often one number is contained in another number, and thus finding *what part* of one number, is another number.

The *dividend* is the number to be divided. The *divisor* is the number by which you divide. The *quotient* is the answer obtained by dividing.

Reduction is changing units of one order, to units of another order.

Reduction ascending, is changing units of a lower, to a higher order.

Reduction descending is changing units of a higher, to a lower order.

Note to Teachers.—*A review* of this *First Part*, will be found more useful than an increased number of examples.

ARITHMETIC.

SECOND PART.

NUMERATION.

Numeration is the art of expressing numbers by *words*, or by *figures*.

Figures are sometimes called *numbers*, because they are used to *represent* numbers. Thus the figure 4, is often called the *number* four, because it is used to represent that number.

There are thirty-five *words*, that are commonly used in numeration; viz.: *one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred, thousand, million, billion, trillion, quadrillion, quintillion, sextillion.*

Those words ending in *teen*, are the words *three, four,* &c. with *teen*, which signifies *and ten*, added to them.

What is the meaning of *fourteen?* Ans. *Four and ten.* What is the meaning of *thirteen?* of *nineteen?* of *seventeen?*

Those ending in *ty*, are the words *two, three, four,* &c. with *ty*, which means *tens*, added to them.

What is the meaning of sixty? of seventy? of eighty? of twenty? of thirty?

The words of spoken numeration would be more uniform, if *eleven* and *twelve*, had been called *oneteen* and *twoteen.*

The Latin and Greek numerals are so often used in the various sciences, that it is important for pupils to learn their names. They are therefore put down with the figures, and the English names. The *figures* are called *Arabic*, because first introduced into Europe from Arabia.

What is numeration? Why are *figures* sometimes called *numbers?* How many *words* are used in numeration, and what are they? What does *teen* signify? What is the meaning of *fourteen?* *seventeen?* *nineteen?* What does *ty* signify? What is the meaning of sixty? seventy? eighty? twenty? thirty? Why are the figures used called *Arabic.*

ENGLISH, LATIN, AND GREEK NUMERALS.

Arabic Figures.	*English Names.*	*Latin Names.*	*Greek Names.*
1	One.	Unus.	Eis.
2	Two.	Duo.	Duo.
3	Three.	Tres.	Treis.
4	Four.	Quatuor.	Tessares.
5	Five.	Quinque.	Pente.
6	Six.	Sex.	Hex.
7	Seven.	Septem.	Hepta.
8	Eight.	Octo.	Okto.
9	Nine.	Novem.	Ennea.
10	Ten.	Decem.	Deka.
11	Eleven.	Undecim.	Endeka.
12	Twelve.	Duodecim.	Dodeka.
13	Thirteen.	Tredecim.	Dekatreis.
14	Fourteen.	Quatuordecim.	Dekatessares.
15	Fifteen.	Quindecim.	Dekapente.
16	Sixteen.	Sexdecim.	Dekaex.
17	Seventeen.	Septendecim.	Dekaepta.
18	Eighteen.	Octodecim.	Dekaocto.
19	Nineteen.	Novemdecim.	Dekaennea.
20	Twenty.	Viginti.	Eikosi.
30	Thirty.	Triginta.	Triakonta.
40	Forty.	Quadraginta.	Tesserakonta.
50	Fifty.	Quinquaginti.	Pentakonta.
60	Sixty.	Sexaginta.	Hexakonta.
70	Seventy.	Septuaginta.	Hebdomekonta.
80	Eighty.	Octoginta.	Ogdoekonta.
90	Ninety.	Nonaginta.	Ennenekonta,
100	Hundred.	Centum.	Hekaton.
1000	Thousand.	Mille.	Chilio.
1000000	Million.		

Billion, Trillion, Quadrillion, Quintillion, Sextillion, &c. are made by adding ciphers to 1.

If any higher number than *sextillion* is to be expressed, the names are made by the Latin numbers, with *illion* added to them ; as *septillion*, *octillion*, &c.

A *unit* has been defined as "a single thing of any kind."

But a unit of one kind, may be *made up* of several units of

How are higher numbers than sextillions expressed? What is a unit?

another kind. Thus the unit *one dollar* is made up of ten units, of the kind, or *order* called *dimes;* and one *dime* is made up of ten units of the order called *cents. Order* means the same as *kind.*

A unit which is of the *most value,* is called a unit of a *higher order.*

Which unit is of the highest order, a *dollar* or a *cent?*

How many units of the order of *dimes,* are there in one unit of the order of *dollars?*

How many units of the order of *mills,* make one unit of the order of *cents?*

How many units of the order of *cents,* make one unit of the order of *dimes?*

Every *figure* expresses a certain *number;* but the number it expresses, depends upon the *order* in which it is placed.

If the figure (2) stands alone, it expresses two *units,* and is said to be in the *first* or *unit* order.

But if it has a figure at the right of it, thus (20) it expresses *two tens,* or *twenty,* and is in the *second order,* or the *order of tens.*

The cipher is put at the right, to make the 2 stand in the order of tens, and to show that there are *no* units of the *unit order.* If *some* figure was not placed there, the 2 would be in the *unit* order.

If the figure 2 has *two* figures at the right of it, thus (200) it represents *two hundreds,* and stands in the *third order,* or the *order of hundreds.*

From this it appears, that in numeration, *the number expressed by any figure, depends upon the order in which it stands.*

The number which any figure expresses when it is considered alone, is called its *simple* value. The number it expresses when placed with other figures, is called its *local value.*

Exercises.

Write *one ten.*—Why is the cipher used? What would the number be, if the cipher were removed?

What distinction is made in regard to units on page 32? What is the meaning of the word *order?* What is meant by a unit of a higher order? What does every figure represent? What does the *number* which any figure represents depend upon? If a figure stands alone, in what order is it? If it has one figure at the right of it, in what order is it? If it has two figures at the right of it, in what order is it? In this number, (234) in what order is the 2? the 3? the 4? What is the *simple,* and what the *local* value of figures? When 2 is considered alone, what is its simple value? When it is written with two figures at the right, what is its local value?

Write *one ten and one unit*. What is the name of this number? Ans. *Eleven*.

Write *one ten and two units*. What is the name of this number?

Write *one ten and three units*. What is the name?

Write *one ten and four units*. What is the name?

Write *one ten and five units*. What is the name?

Write *one ten and six units*. What is the name?

Write *one ten and seven units*. What is the name?

Write *one ten and eight units*. What is the name?

Write *one ten and nine units*. What is the name?

Write *two tens*. What is the name? Ans. *Twenty*.

Write *three tens*. What is the name?

Write *four tens; five tens; six tens; seven tens; eight tens; nine tens;* and tell their names.

Write *one* of the order of *hundreds*.

Write *two* of the order of *hundreds; one* of the order of *tens;* and *four* of the order of *units*.

Write two of the order of hundreds; no tens; four units.

Write four hundreds, no tens, no units.

Write two hundreds, eight tens, and nine units. Seven hundreds, six tens, and three units. Two tens, and two units. Nine tens, and six units. Four hundreds, six tens, and four units. Five hundreds, five tens, and five units. Nine hundreds, seven tens, and three units. Four hundreds, eight tens, and four units. Eight hundreds, nine tens, and nine units. Two hundreds, six tens, and three units. One hundred, two tens, and three units. Two hundreds, five tens, and seven units. One ten, and three units. Seven tens, and three units. Nine hundreds, nine tens, and nine units.

In reading numbers, we can either mention *each order separately*, or simply mention the *names* of the numbers.

Thus we can call this number, (21) either *two tens, and one unit*, or *twenty-one*.

This number (305) can be read, 3 hundreds; 0 tens; 5 units; or it can be called *three hundred and five*.

The following numbers are read both ways, thus:

10 One ten; no units; or *ten*.

11 One ten; one unit; or *eleven*.

208 Two hundreds; no tens; eight units; or *two hundred and eight*.

What two ways of reading numbers are there? Give an example.

40 Four tens, no units; or *forty*.

Let the pupil read the following numbers both ways.

111. 203. 41. 37. 542. 1. 11. 12. 60. 300. 101. 639. 700. 305.

In this number, (203) why is the cipher put in? What would the number be if it were left out?

In numeration, every unit of one order, is considered as composed of *ten units* of a lower order; just as in the coins of this country, ten units of the order of *cents*, make one unit of the order of *dimes*, and ten units of the order of dimes, make one unit of the order of *dollars*.

So in numeration, ten units of the *order of units*, make *one ten;* ten units of the *order of tens*, make one unit of the order of *hundreds;* ten *hundreds* make one unit of the order of *thousands;* ten thousands make *one* of the order of *tens of thousands;* ten *tens of thousands*, make *one* of the order of *hundreds of thousands;* ten *hundreds of thousands*, make *one* of the order of *millions*, &c.

Wherever there are *nine* units of any order, if there is *another* added, the number becomes *one unit* of the next higher order.

If we had nine cents, and should add another, instead of calling the amount ten cents, we could call it *one dime;* and so when ten units are added together, we can call them *one unit* of the order of *tens*, instead of *ten units* of the *unit order;* and when we have ten units of the order of *tens*, we can call them *one unit* of the order of *hundreds*.

EXERCISES.

If nine cents have one more added, in what order do they become a unit?

If nine dimes have another added, in what order do they become a unit?

Ten units of the order of dollars, make one unit of what order?

Ten tens, make one unit of what order?

Ten units, make one unit of what order?

Ten hundreds make one unit of what order?

The following are the *names* of the *orders*.

First order,	Units.
Second order,	Tens.

How many units of one order make one unit of a higher order? If one unit is added to nine units of any order, what do they become?

Third order,	Hundreds.
Fourth order,	Thousands.
Fifth order,	Tens of thousands.
Sixth order,	Hundreds of thousands.
Seventh order,	Millions.
Eighth order,	Tens of millions.
Ninth order,	Hundreds of millions.
Tenth order,	Billions.
Eleventh order,	Tens of Billions.
Twelfth order,	Hundreds of Billions.
Thirteenth order,	Trillions.
Fourteenth order,	Tens of trillions.
Fifteenth order,	Hundreds of Trillions.
Sixteenth order,	Quadrillions.
Seventeenth order,	Tens of Quadrillions.
Eighteenth order,	Hundreds of Quadrillions.
Nineteenth order,	Quintillions.
Twentieth order,	Tens of Quintillions.
Twenty-first order,	Hundreds of Quintillions.
Twenty-second order,	Sextillions.

Sextillions are as high as there is ordinarily any need of writing or reading.

In all the above orders, "*Ten units of one order, make one unit of the next higher order.*"

Let the pupil write the following

Exercises.

1. Five units.
2. Three tens; two units.
3. Thirty-two.
4. Three and ten, or *thirteen.*
5. Four and ten.
6. Four tens, or *forty.*
7. Six and ten.
8. Six tens.
9. Sixteen.
10. Sixty.
11. One hundred and sixteen.
12. One hundred, one ten, and six.
13. One hundred and sixty.
14. One hundred, and six tens.
15. Two hundred, two tens.
16. Two hundred and twenty.
17. Two hundred and thirty.
18. Two tens and two units.
19. Twenty-two.
20. Two hundreds and two units.
21. Five tens and two units.
22. Five hundreds.
23. Five tens.
24. Fifty.
25. Five hundred, and five units.
26. Five and ten.

What are the names of the orders? If a figure 2 stands in the *first* order, what *number* does it express? What number does it express, if it stands in the *fourth* order? In the *second* order? In the *fifth* order? In the *sixth? seventh? eighth?* How many units of one order make one unit of the next higher order?

27. Fifteen.
28. Fifty-seven.
29. Four hundreds, six tens.
30. Four hundred and sixteen.
31. Four hundreds, one ten, and six.
32. Four hundred, and six.
33. Two hundred and sixty-six.
34. Three hundred, ten, and one.
35. Three hundred and eleven.
36. Three hundred, ten, and two.
37. Three hundred and twelve.
38. Four hundred and one.
39. One hundred and forty-two.
40. Two hundreds, two tens.

Let the pupil write the following

EXERCISES.

1. One unit of the fourth order. What number is it? Which orders have ciphers in them?

2. Two units of the fourth order; one unit of the second order, and one unit of the first order. What number is it? What order has a cipher in it?

3. Two thousands; one hundred; five tens; six units.

4. Twenty-one hundreds; five tens; six units.

Is there any difference between the two last numbers?

5. Three thousands, four hundreds, six tens and three units.

6. Thirty-four hundred, and sixty-three.

Is there any difference in the two last numbers?

7. Three thousands and three units.

Which orders have ciphers placed in them?

8. Three thousands, six hundreds.

Which orders have ciphers placed in them?

9. Thirty-six hundred.

What two ways of reading this last number?

10. Twenty thousand.

11. Two tens of thousands.

Is there any difference between these two last numbers?

12. Twenty-four thousand.

What two ways of reading this last number?

13. One hundred thousand, two tens of thousands, five thousands, six hundreds, four tens, and three units.

14. One hundred and twenty-five thousand, six hundred and forty-three.

Is there any difference between these two last numbers?

15. Two tens of thousands, one thousand, four hundreds, six tens, five units.

What two ways of reading this number?

16. Four hundred and sixty-two thousand, five hundred and six.

What two ways of reading this last?

17. Forty-four thousand, four hundred and forty-four.

What two ways of reading this last?

18. Four hundreds of thousands, five thousands, six hundreds, two tens, five units.

What two ways of reading this last number?

19. Two hundred thousand, two thousand, two units.

What orders have ciphers placed in them?

20. Twenty thousand, and two units.
21. Two hundred and six thousands, four hundred and six.
22. Sixty-four thousand and three.
23. Sixteen thousand.
24. Fourteen thousand and seven.
25. Five tens of thousands, and six units.
26. Two hundreds of thousands, two hundreds, two units.
27. Two hundred and sixty-four thousand, and six.
28. Four thousand, and five units.
29. One hundred thousand, and three.
30. Sixteen thousand, six hundred and six.
31. Twenty-four thousand and three.

In order to read and write large numbers more conveniently, they are divided into *periods* of *three* figures each, by means of commas, thus:

876,469,764,256,622,895,946,852.

The *first* right hand period is called the *unit period;* and contains the *orders* called *units*, *tens*, and *hundreds*.

The *second period*, is called the *thousand period;* and contains the *orders* called *thousands*, *tens of thousands*, and *hundreds of thousands*.

The *third period* is called the *million period*, and contains the *orders* called *millions*, *tens of millions*, and *hundreds of millions*.

The *fourth period* is called the *billion period;* and contains the *orders* called *billions*, *tens of billions*, and *hundreds of billions*.

The *fifth period* is called the *trillion period;* and contains the *orders* called *trillions*, *tens of trillions*, and *hundreds of trillions*.

The *sixth period* is called the *quadrillion period;* and contains the *orders* called *quadrillions*, *tens of quadrillions*, and *hundreds of quadrillions*.

The *seventh period* is called the *quintillion period;* and contains the *orders* called *quintillions*, *tens of quintillions*, and *hundreds of quintillions*.

The *eighth period* is the *sextillion*.

The following are the periods which must be learned in *succession*, beginning with the *highest*, as well as with the *lowest*; thus,

First Period, *Unit.*	Eighth Period, *Sextillion.*
Second Period, *Thousand.*	Seventh Period, *Quintillion.*
Third Period, *Million.*	Sixth Period, *Quadrillion.*
Fourth Period, *Billion.*	Fifth Period, *Trillion.*
Fifth Period, *Trillion.*	Fourth Period, *Billion.*
Sixth Period, *Quadrillion.*	Third Period, *Million.*
Seventh Period, *Quintillion.*	Second Period, *Thousand.*
Eighth Period, *Sextillion.*	First Period, *Unit.*

What is the first period? the third? the fifth? the second? the fourth? the seventh? the sixth? the eighth?

The pupil may write the names over the periods until accustomed to reading them; thus,

Tril.	Bil.	Mil.	Thous.	Units.
32	427	983	254	693

The above may be read in the following manner:

The first *left hand* period is read, 3 tens of trillions; 2 units of trillions; or *thirty-two trillions.*

The next period is read, 4 hundreds of billions; 2 tens of billions; 7 units of billions; or *four hundred and twenty-seven billions.*

The next period is read, 9 hundreds of millions; 8 tens of millions; 3 units of millions; or *nine hundred and eighty-three millions.*

The next period is read, 2 hundreds of thousands; 5 tens of thousands; 4 units of thousands; or *two hundred and fifty-four thousand.*

The next period is read, 6 hundreds; 9 tens; 3 units; or *six hundred and ninety-three.*

The following is a number in which several orders are omitted, having *ciphers* in place of numbers.

Quin.	Quad.	Tril.	Bil.	Mil.	Th.	U.
33	067	004	803	064	000	400

What are the names of the *eight periods*, and what *orders* does each period contain? Repeat the periods beginning at the lowest or unit period. Repeat them beginning at the highest. Read the above number in the two different ways.

Let the pupil first tell what *periods* and what *orders* are omitted, having ciphers instead of numbers.

The above number may be read thus:

Begin at the left and read; 3 tens of quintillions, and 3 units of quintillions; or *thirty-three quintillions.*

The next period is, no hundreds of quadrillions; 6 tens of quadrillions; and seven units of quadrillions; or *sixty-seven quadrillions.*

The next period is, no hundreds of trillions; no tens of trillions; 4 units of trillions; or *four trillions.*

The next period is, 8 hundreds of billions; no tens of billions; 3 units of billions; or *eight hundred and three billions.*

The next period is, no hundreds of millions; 6 tens of millions; 4 units of millions; or *sixty-four millions.*

The next period, as it has no hundreds, tens, or units of thousands, may be omitted entirely, *when reading.*

The next period is, 4 hundreds; no tens; no units; or *four hundred.*

The best and most common way of reading, is that in the *italics*, and then all together, it reads thus:

Thirty-three quintillion; sixty-seven quadrillion; four trillion; eight hundred and three billion; sixty-four million; four hundred.

Let the pupil read the following sum in both ways:

Quin.	Quad.	Tril.	Bil.	Mil.	Th.	Un.
607	300	000	763	490	068	002

RULE FOR READING WHOLE NUMBERS.

Point off into periods of three figures each, beginning at the right. Read each period as if it stood alone, and then add the name of the period.

NOTE.—When a period or order is omitted, it is not necessary to mention it at all.

Before reading, let the pupil tell what *periods* and *orders* are omitted, and represented by ciphers.

Let the pupil point off, and read the following figures:

Read the above numbers in the two different ways. What is the rule for reading whole numbers? In the above numbers what *periods* and orders are omitted?

1	2	31	304	300046	200200200
111	24	40	600	300005	2030003000
100	136	400	611	1200437	311001300
101	3024	4040	693	1200039	60009090
1011	2002	6000	4004	4960004	100100001
2002	46900	40640	103006	1430096	2071113603
3041	60021	600003	1063007	6000007	1000673
201	62003	100014	103964	86004369	101700013
2010	6040064	600436	140001	20064000	600040006
3004	46923	64003	400006	400400400	300010000

227034293	9623000062
200004900	10043259054
3690200000	43600078609
30006340200	459643723007
602030004296	6129420040000040367
40000643209437	3907650060042300000
2376000964300600000	3967700005439650000076

It is necessary for the pupil to understand, that the French and English arithmeticians use different methods of numeration.

The English have their periods contain *six* orders, and the French only *three*.

This makes no difference till we come to ***hundreds of millions.*** After that, it makes a great difference, as will be seen by the following comparison.

It must be noticed, that the same figures are used in both.

English Method.

Trillions.	Billions.	Millions.	Units.
579364,	028635,	419763,	215468.

French Method.

Sext.	Quin.	Qua.	Trill.	Bill.	Mill.	Th.	Units.
579,	364,	028,	635,	419,	763,	215,	468.

From the above it can be seen, that all the orders above ***hundreds of millions***, in both methods, give the ***same name***, to a very different value.

Thus, the orders of ***thousands of millions***, ***tens of thousands of***

What is the difference between the English and French method of numerating? Where does it make a difference, and where does it not? How would a *billion*, in the English method, be read in the French? How would *one hundred billion*, in the English method, be read in the French? How would *one billion*, in the French method, be read in the English?

millions, and *hundreds of thousands of millions*, in the *English method*, would be read as *billions*, *tens of billions*, *and hundreds of billions*, in the French method.

Billions, tens of billions, and hundreds of billions, in the English method, are equivalent to trillions, tens of trillions, and hundreds of trillions, in the French method.

Five *trillion*, in the *French* method, would be read five *billion*, in the English; and five *trillion*, in the English method, would be read five *quadrillion* in the French.

The French method is adopted in this work, because it is both the most convenient, and the most common. But the pupil needs to understand the difference between the two modes, and the teacher should make the class point off and read numbers by both.

Point off and read the following numbers, first by the French, and then by the English method.

765432176500431	32698000000040000360093
9870000654321765432	436789643645964379629364

In order to write numbers correctly, the pupil must learn thoroughly, the *succession* of the periods *beginning at the left*.

Thus, *Sextillion*, *Quintillion*, *Quadrillion*, *Trillion*, *Billion*, *Thousand* and *Unit*.

RULE FOR WRITING WHOLE NUMBERS.

Begin with the highest period, and write first the hundreds, then the tens, and then the units of that period. Proceed thus, until all the periods are written. Place a comma between each period. If any period or order is omitted, place ciphers in its place.

NOTE.—Ciphers *prefixed* to a *whole number*, have no effect upon the value. A number, therefore, should never be begun with a cipher.

EXERCISES.

Write two thousands and two. What orders are omitted?

Write two millions, two thousands, and four. What orders are omitted?

How would *six hundred billion* in the French method, be read in the English? Which method is adopted in this work? What is the succession of the periods beginning at the left? What is the rule for writing whole numbers? Why should not any whole number, begin with a cipher?

Write three hundred and twenty-four. What period and orders in this number?

Write two hundred thousands and four. What orders are omitted in this last number?

Write two millions and six? What *period* omitted? What *orders* omitted?

Write six millions; two hundred and three. Which period and what orders are omitted?

Write twenty-four millions; three hundred. Which period and what orders are omitted?

Write the following sums, and mention the periods and orders which are omitted.

EXERCISES.

1. One billion; twenty-four millions; three thousands and three.
2. Four hundred and sixty-nine billions; forty-four thousands; and seventeen.
3. Fifty billions; three hundred millions; four hundred and fifty thousands; and nineteen.
4. Fifty billions, and seven.
5. Four hundred and thirteen millions, and two thousand.
6. Nineteen billions, and one million.
7. Six trillions; nine thousands, and ten.
8. Seven trillions; nineteen billions; ten thousands, and four hundred.
9. Four hundred and nine trillions; sixteen millions; eleven thousands and forty.
10. Fifteen billions; two hundred and four millions; six thousands, and twenty-one.
11. Sixty-four millions; four hundred thousands; three hundreds.
12. Sixteen millions; five hundred thousands, and six.
13. Three trillions; fourteen millions; seven thousands.
14. Two hundred and sixteen millions.
15. Two billions; sixteen millions, and sixteen.
16. Three hundred and six trillions; four thousands, and six.
17. Two quintillions; six quadrillions and five.
18. Three hundred and sixty-four thousands.
19. Three millions and six.
20. Fourteen trillions; three hundreds.
21. Sixteen trillions; four millions; two hundred and four thousands; seven hundred and one.

22. Three sextillions; one hundred quadrillions; fourteen trillions; two hundred and sixty billions; four hundred millions; sixteen thousands; four hundred and one.

23. Five millions; two hundred thousands, and sixty-two.

24. Two hundred and five millions, and seventy-four.

25. Two hundred and six billions; four millions, and six thousands.

26. Two hundred sextillions; four hundred millions; three hundred and four thousands; two hundred and six.

27. Fifteen quintillions; six quadrillions; one hundred trillions; forty-four billions; two millions, and forty nine.

28. Fifty quadrillions; six hundred trillions; forty-three millions; two thousands four hundred and six.

29. Two hundred and six trillions; forty-three billions; four hundred and nine millions; sixty-four thousands; four hundred and ninety-six.

30. One hundred and four billions; six millions; forty-nine thousands; four hundred and ninety-six.

31. Thirteen millions; four hundred thousands; six hundred and forty-nine.

32. Six sextillions; five quintillions; four quadrillions; three trillions; two billions; and one million.

NUMERATION OF VULGAR FRACTIONS.

The figures used in numeration are of two kinds,—Figures for a number of *whole things*, and figures for a number of *parts of things*.

A *unit* is a *whole thing* of any kind.

A *fraction* is a *part* of *one thing*; or a *part* of *several things*.

Figures may therefore be divided into *fractional* and *unit* figures.

The following is the mode of showing when the numbers represented are several *whole* things, and when they are several *parts* of things.

When there are two *whole* things, their number is expressed thus, (2). This is called a *unit figure*.

How many kinds of figures are used in numeration? What are they? What are units and fractions? What distinction is made between units and fractions on page 32? When two *whole* things are expressed, what figure is used?

But if a whole thing is divided into *three parts*, and we wish to express *two* of these, by figures, we write one figure, to show *into how many* parts the whole thing is divided, and then above it, write the *number of parts* we wish to express; thus, ($\frac{2}{3}$).

This is called a *fractional figure*. The *lower* figure shows into how many parts the whole thing is divided, and the *upper* figure shows how many of these parts are expressed.

In $\frac{3}{8}$, into how many parts is the whole thing divided, and how many of these parts are expressed?

In $\frac{4}{5}$, into how many parts is the whole thing divided, and how many parts are expressed?

In $\frac{6}{7}$? In $\frac{4}{5}$? In $\frac{7}{9}$? In $\frac{6}{8}$? In $\frac{3}{12}$? In $\frac{4}{20}$?

Fractional figures show into how many parts *one* whole thing is divided, and how many of these parts are expressed. Besides this, they can show *what part* is taken from *several whole things*. Thus $\frac{3}{4}$ shows that one thing is divided into *four parts*, and *three* of them are taken; or that *three whole things*, have a *fourth* taken from *each* of them. For, *three fourths* of *one whole thing*, is the same quantity as *one fourth* of *three whole things*.

If you have *three* apples, and take *one fourth* out of each, how much will you have, and how will you express it in figures? If you divide one apple into *four* parts, and take *three* of these parts, how do you express the quantity taken?

If you have two apples, and take one sixth from each, how much will you have, and how will you express it in figures?

If you divide an apple into six parts, and take two of these parts, how much will you have, and how will you express it in figures?

If an apple is divided into eight parts, and you take three of them, how much will you have, and how will you express it in figures?

If the fraction is considered as showing how *many parts* are taken from *one* unit, then the lower figure shows *into how many parts* a unit is divided, and the upper figure shows *how many of these parts* are taken. But if the fraction is considered as

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If a thing is divided into three parts, how do you express *two* of those parts? In a *fractional figure* what does the lower figure show, and what the upper? What else do fractional figures show? If a fraction shows how many parts are taken from *one* unit, what does the upper figure and what does the lower figure show?

showing *what part* is taken out of *several units*, then the upper figure shows the *number of units*, and the lower figure shows *what part is taken from each.*

Thus the fraction $\frac{2}{6}$ may be considered as expressing, ***two sixths*** of *one thing*, or as *one sixth* of *two* things.

$\frac{3}{12}$ is either *one twelfth* of *three things*, or *three twelfths* of *one thing.*

$\frac{4}{5}$ is either *four fifths* of *one thing*, or *one fifth* of *four things.*

$\frac{2}{9}$ either shows that *one ninth* is taken out of *two* things; or that *two ninths* are taken out of *one thing.*

If $\frac{4}{7}$ is considered as showing *how many parts* are taken out of *one thing* it is *four sevenths of one unit.* If it is considered as showing *what part* is taken out of several things, it is *one seventh of four units.*

If $\frac{2}{3}$ shows *how many parts* are taken out of *one thing*, it is *two thirds of one thing.* If it shows *what part* is taken out of *several things*, it is *one third of two things.*

If $\frac{7}{8}$ is considered as showing *how many parts* are taken out of *one unit*, what does the 8 show, and what does the 7 show?

If it is considered as expressing *what part* is taken out of *several units*, what does the 7 show, and what does the 8 show?

If $\frac{4}{6}$ is considered as expressing *how many parts* are taken out of *one unit*, what does the 6 show, and what does the 4 show? If it is considered as expressing *what part* is taken out of *several units*, what does the 4 show, and what does the 6 show?

Whenever the *numerator* is *larger* than the *denominator*, the fraction is called an *improper fraction*, and always is to be considered as expressing *what part* is taken out of *several units.*

Which of the following are improper fractions?

$\frac{6}{3}$ $\frac{7}{4}$ $\frac{8}{14}$ $\frac{9}{6}$ $\frac{12}{2}$ $\frac{14}{5}$ $\frac{6}{8}$ $\frac{4}{7}$ $\frac{8}{3}$.

RULE FOR READING VULGAR FRACTIONS.

Read the number of parts expressed by the numerator, and then the size of the parts expressed by the denominator; or *

* The pupils should give an example, when reciting these rules, to show that they understand them.

Give an example. If a fraction shows *what part* is taken from *several* units, what does the upper and what does the lower figure show? What is an improper fraction, and what does it express?

*Read the part expressed by the denominator, and then the number of units, expressed by the numerator.**

Read the following fractions in both ways, thus: $\frac{3}{4}$ is either *three fourths of one thing, or one fourth of three things.*

$\frac{3}{5}$ is either *three fifths of one*, or *one fifth* of *three.*

$\frac{2}{3}$ $\frac{4}{6}$ $\frac{6}{9}$ $\frac{2}{6}$ $\frac{12}{13}$ $\frac{5}{10}$ $\frac{6}{12}$ $\frac{9}{18}$.

RULE FOR WRITING VULGAR FRACTIONS.

*Write the number of parts into which a unit is divided, and draw a line above it. Over it write the number of parts which are to be expressed; or **

*Write the whole numbers which have a certain part taken from them, and draw a line under. Beneath it write the figure which expresses the part which is to be taken out of each of the units above.**

Let the pupil write the following

EXERCISES.

If a man divided an apple into eight parts, and gave away *five* of these parts, how do you express the quantity he gave away, and the quantity he kept?

If a man had *three apples*, and cut out a fourth part of each, and gave it away, how do you express what he gave away?

If a man had twelve oranges, and one sixth of each was decayed, how do you express the quantity of decayed oranges he had?

If a man had five casks of wine, and a twelfth part leaked out of each, how do you express what he lost?

To Teachers. Various exercises like the above are necessary.

DECIMAL NUMERATION.

There is another mode of writing fractions, in which the *numerator* only is written. The denominator, although not written, is always *understood* to be 1, and a certain number of *ciphers*.

These fractions are called *Decimals*.

Thus in writing decimals, if we are to express *two tenths*, in-

What are the rules for reading vulgar fractions? What are the rules for writing vulgar fractions?

stead of writing it thus, $\frac{2}{10}$, the *numerator* only is written, and a comma, called a *separatrix*, is placed before it, thus ,2.

The following is the rule, by which it is known what is the *denominator*.

The denominator of a decimal is always one, and as many ciphers as there are figures in the numerator, or decimal.

What is the denominator of this decimal, ,2?

Ans. 1 and one cipher.

How many ciphers in the denominators of these decimals, ,34. ,600. ,3246. ,56945. ,3694.?

If the decimal has *one* figure, it expresses *tenths*. Thus ,2 is two *tenths*.

If it has *two* figures, it expresses *hundredths*. Thus ,02 is two *hundredths*.

If it has *three* figures, it expresses *thousandths*. Thus ,002 is two *thousandths*.

If it has *four* figures, it expresses *tenths of thousandths*. Thus ,0002 is two *tenths of thousandths*.

If it has *five* figures, it expresses *hundredths of thousandths*. Thus ,00002 is two *hundredths of thousandths*.

What does this decimal express, ,3?

Ans. Three *tenths*.

What does this decimal express, ,30?

Ans. Thirty *hundredths*.

What does this decimal express, ,003?

Ans. Three *thousandths*.

What does this decimal express, ,0003?

Ans. Three *tenths of thousandths*.

What does this decimal express, ,5? Ans. 5 *tenths*.

What does this decimal express, ,15? Ans. Fifteen *hundredths*.

What does this decimal express, ,110?

What does this decimal express, ,2000?

What does this decimal express, ,00002?

Ans. Two *hundredths of thousandths*.

A decimal must always have the number of *figures* in the numerator, equal to the number of *ciphers* in the denominator;

How do decimal fractions differ from vulgar? What is the rule for ascertaining the denominator of a decimal? What must the number of figures in the numerator equal?

therefore it is necessary to *learn* how many ciphers there are in each kind of denominator.

If the decimal is *tenths*, there is *one cipher* in the denominator; if *hundredths*, there are *two* ciphers; if *thousandths*, there are *three* ciphers; if *tenths of thousandths*, there are *four* ciphers; if *hundredths of thousandths*, there are *five* ciphers, &c.

Of course in writing decimals, if *tenths* are to be expressed, there must be only *one* figure in the *numerator*, or decimal; if *hundredths*, there must be *two* figures; if *thousandths*, there must be *three* figures; if *tenths of thousandths*, there must be *four* figures; if *hundredths of thousandths*, there must be *five* figures, &c.

EXERCISES.

If you are to write two *tenths*, how many *figures* must there be in the numerator or decimal, and how many ciphers are understood to be in the denominator? Write two tenths. (,2.)

If you are to write *two hundredths*, how many ciphers are understood to be in the denominator, and how many figures must there be in the numerator?

Write, *two hundredths*.

In writing this last, the pupil must first write the 2, and then as there must be as many figures in the numerator, as there are ciphers in the denominator, a *cipher* is placed *before* the 2, and then the separatrix is prefixed thus, ,02.

If the cipher were placed *after* the 2, how would it read?

Ans. *Twenty* hundredths, instead of *two* hundredths.

If the cipher were not placed before the 2, how would it read? Ans. Two *tenths*.

If another cipher is placed before the ,02 thus, ,002 how does it read?

What does the denominator express, when there are *three* figures in the decimal. Ans. *Thousandths*.

What does it express when there are *four* figures in the decimal?

Let the pupil write the following.*

1. Two *tenths*. Two *tens*.
2. Two *hundredths*. Two *hundreds*.
3. Two *thousandths*. Two *thousands*.
4. Two *tenths of thousandths*. Two *tens of thousands*.

* The exercises in *whole numbers* are added, that pupils may notice the difference.

5. Two *hundredths of thousandths.* Two *hundreds of thousands.*

6. Five *tenths.* Five *tens.*

7. Fifteen *hundredths.* Fifteen *hundreds.*

8. Fifteen *thousandths.* Fifteen *thousands.*

9. Fifteen *tenths of thousandths.* Fifteen *tens of thousands.*

10. Fifteen *hundredths of thousandths.* Fifteen *hundreds of thousands.*

11. One *tenth.* One *ten.*

12. Eleven *hundredths* Eleven *hundreds.*

13. One hundred and fifteen *thousandths.* One hundred and fifteen *thousands.*

14. Five *tenths.* Five *tens.*

15. Fifty-five *hundredths.* Fifty-five *hundreds.*

16. Five hundred *thousandths.* Five hundred *thousands.*

17. Five hundred and five *thousandths.* Five hundred and five *thousands.*

18. Fifteen *thousandths.* Fifteen *thousands.*

19. Five *thousandths.* Five *thousands.*

20 Two hundred *thousandths.* Two hundred *thousands,*

21. Twenty-nine *thousandths.* Twenty-nine *thousands.*

22. Five *hundredths.* Five *hundreds.*

23. Forty *hundredths.* Forty hundreds or four *thousands.*

24. Nine *tenths of thousandths.*

25. Nineteen *tenths of thousandths.*

26. Nine hundred *tenths of thousandths.*

27. Two thousand *tenths of thousandths.*

28. Two thousand and two *tenths of thousandths.*

29. Three thousand three hundred *tenths of thousandths,*

30. Thirty-two hundred *tenths of thousandths.*

31. Six *tenths of thousandths.*

32. Four *hundredths of thousandths.*

33. Fourteen *hundredths of thousandths.*

34. Four hundred *hundredths of thousandths.*

35. Two thousand and six *hundredths of thousandths.*

36. Sixty-four thousand *hundredths of thousandths.*

37. Sixteen thousand and four *hundredths of thousandths,*

38. Four thousand and nine *hundredths of thousandths.*

39. Six *hundredths of thousandths.*

40. Five thousand and four *hundredths of thousandths.*

41. Sixty-five thousand *hundredths of thousandths.*

42. Nine hundred and one *hundredths of thousandths.*
43. Twenty-nine hundred *hundredths of thousandths.*
44. Twelve *tenths of thousandths.*
45. Fifteen *hundredths.*
46. Sixty-four *thousandths.*
47. Nine hundred and one *tenths of thousandths.*

Decimals can be read in two different ways.

Thus, ,21 can be read, either as *two tenths*, and *one hundredth;* or as *twenty-one hundredths.*

This can best be illustrated, by the coin of the United States. Thus, 2 dimes, 1 cent, can be read, either as *twenty-one cents,* or as *two dimes* and *one cent.*

Thus again, 1 dollar, 3 dimes, and 2 cents, can be called, either 132 cents; or 13 dimes, 2 cents; or 1 dollar, 3 dimes, and 2 cents.

In like manner, *decimals* may be read in different ways. Thus, 234 can be read either as 234 *thousandths;* or 2 tenths, 3 hundredths, and 4 thousandths; or 23 hundredths, and 4 thousandths; or 2 tenths, and 34 thousandths.

NOTE. Let the teacher illustrate the above with coin.

EXERCISES.

Write two *tenths.*

Write twenty *hundredths.*

,2 is how many *hundredths?*

Ans. There are *ten times* as many *hundredths* as there are *tenths* in a thing. Therefore ,2 is ten times as many *hundredths,* or 20.

Is there any difference in the *value* of ,2 and ,20? What *is the difference* between them?

Ans. The ,20 has ten times *more pieces,* and each piece is ten times *smaller* than the ,2; but there is no difference in the *value.*

,3 is how many hundredths? ,4 is how many hundredths?

,30 is how many tenths? ,40 is how many tenths?

Write two tenths, and four hundredths. In this sum how many *hundredths?*

Write thirty-four *hundredths.* In this sum how many *tenths?*

Write 2 tenths, 6 hundredths, or *twenty-six hundredths.*

Write 4 tenths, 9 hundredths, and read it both ways.

Write 6 tenths, 7 hundredths, 5 thousandths, or *six hundred and seventy-five thousandths.*

Write 6 tenths, 4 hundredths, and 5 thousandths.

Write nine tenths, six hundredths, and six thousandths, and read them both ways.

Write seven tenths, six hundredths, five thousandths, and nine tenths of thousandths, and read them both ways.

Write nine tenths, no hundredths, six thousandths, no tenths of thousandths, and five hundredths of thousandths, and read it both ways.

Write six tenths, no hundredths, no thousandths, and five tenths of thousandths, and read it both ways.

Write six thousand four hundred and thirty-six, *tenths of thousandths*, and tell how many tenths, hundredths, and thousandths there are.

Write four hundred and seventy-nine *thousandths*, and tell how many tenths, and hundredths there are.

Write five hundred and six *thousandths*, and tell how many tenths there are.

Write five hundred and ninety-six *hundredths of thousandths*, and read it both ways.

From the above it appears, that in decimals, the order *next to the separatrix* is *tenths;* the *second* order from the separatrix is *hundredths;* the *third* order is *thousandths;* the *fourth* order is *tenths of thousandths;* the *fifth* order is *hundredths of thousandths*, &c.

Decimals are often written with whole numbers. Thus, 2,5, 36,349.

Whole numbers and decimals together, are called *mixed decimals.* When decimals are written without whole numbers, they are called *pure decimals.*

Write twenty-four whole numbers, and twenty-four *hundredths.* Two hundred whole numbers, and five *tenths.*

RULE FOR READING DECIMALS.

Read the numerator, as if it were whole numbers, and then add the name of the denominator; or, Read the number of each separate order, and follow it with the name of the order in which it stands.

In decimals what is the first order, at the right of the separatrix? What is the *second* order? What is the *fourth* order? What is the third? The fifth? What are mixed and pure decimals? What are the rules for reading decimals?

Read the following decimals both ways.

,11. ,020: ,5005. ,32568. ,0505. ,521. ,43002. 24,690. 6,40043. 6,4000. 69,964. 86,0092. 2,002. 16,00020.

In writing decimals from the dictation of the teacher, the pupil needs to understand the *two methods* very clearly.

Thus for example, he may have this decimal, ,00205, dictated in two ways, viz.: 205 *hundredths of thousandths*, or 2 thousandths, and 5 hundredths of thousandths.

In the first mode of dictation, he must write the 205 as if it were whole numbers, and then prefix ciphers to make the figures of the numerator equal to the ciphers of the denominator.

In the second mode of dictation, he must put a cipher in each order which is *not mentioned;* viz.: in the orders *tenths, hundredths*, and *tenths of thousandths*, and a 2 in the order of *thousandths*, and a 5 in the order of *hundredths of thousandths.*

Let the pupil write the following in both methods of dictation.

8 hundredths, 6 tenths of thousandths; or 806 tenths of thousandths.

2 tenths, 4 tenths of thousandths; or 2004 tenths of thousandths.

2 thousandths, 5 tenths of thousandths; or 25 tenths of thousandths.

3 hundredths, 6 thousandths, 5 tenths of thousandths; or 365 tenths of thousandths.

RULE FOR WRITING DECIMALS.

Write the numerator as if it were whole numbers, and then prefix a separatrix. If the figures of the decimal do not equal in number the ciphers of the denominator, prefix ciphers to make them equal, before placing the separatrix; or,

Write each order separately, placing ciphers in the orders omitted.

Write the following:

1. Two hundred and ten *thousandths.*

2. Two tenths, five thousandths, six tenths of thousandths. Here the order of *hundredths* is omitted, and has a cipher put in it.

Give examples of both methods. What are the rules for writing decimals? Give examples of both methods.

3. Two hundred and four *hundredths of thousandths.*

4. Two thousandths; four hundredths of thousandths. What orders are omitted?

5. Sixteen *tenths of thousandths.*

6. One thousandth, six tenths of thousandths. What orders are omitted?

7. Four hundred and five *thousandths.* What orders are omitted?

8. Four tenths, five thousandths. What orders are omitted?

9. Three hundred and sixty-five *tenths of thousandths.* What order has a cipher placed in it?

10. Four hundredths, five tenths of thousandths. What orders are omitted?

11. Twenty-six thousand, nine hundred and forty-six *hundredths of thousandths.*

12. Two tenths, six hundredths, nine thousandths, four tenths of thousandths, six hundredths of thousandths.

In mixed decimals, it will be seen, that the orders are reckoned from the separatrix, *both ways.*

Thus in 98423,46795, the *first* order at the *right* of the separatrix is *tenths*, and the *first* order at the *left* is *units.*

What is the *second order* at the right, and the *second order* at the left of the separatrix?

What is the *third order* at the right, and at the left of the separatrix?

What is the *fourth order* at the right, and at the left of the separatrix?

What is the *fifth order* at the right, and at the left of the separatrix?

If you have the decimal ,2, and place a cipher at the *right*, thus ,20, what does it become? Is the *value* altered? How is it altered?

Ans. The parts are made *ten times smaller*, and there are *ten times more of them*, so that the value remains the same.

If you place a cipher at the *left* of ,2, thus, ,02, what does it become? How much smaller is a hundredth than a tenth?

How much smaller does it make a decimal to *prefix* a cipher to it?

If you put *two* ciphers at the *right* of ,2, what effect is produced? If you put them at the *left* of it, what effect is produced?

The following principle is exhibited above:

Ciphers placed at the right of decimals, change their names, but not their value.

Ciphers placed at the left of decimals, diminish their value ten times for every cipher thus prefixed.

Prefix a cipher to ,91 and read it. Annex a cipher to ,91 and read it.

Prefix a cipher to ,20 and read it. Annex a cipher to ,20 and read it.

SIGNS AND ABBREVIATIONS USED IN ARITHMETIC.

The following signs are used instead of the words they represent.

+ signifies *plus* or *added to.*	*E.* signifies Eagles.
— signifies *minus* or *lessened by.*	$ signifies Dollars.
× signifies *multiplied by.*	*d.* signifies Dimes.
÷ signifies *divided by.*	*cts.* signifies Cents.
= signifies *equals.*	*m.* signifies Mills.

ADDITION.

Addition is uniting several numbers in one.

There are four different processes of addition.

The *first* is *Simple Addition*, in which *ten* units of one order make *one* unit of the next higher order. Thus, ten units make one of the order of tens; ten *tens* make one of the order of hundreds; ten *hundreds* make one of the order of thousands, &c.

The *second* is *Decimal Addition*, in which *decimal fractions* are added to each other. Thus, ,5 ,50 ,505 are added together.

The *third* is *Compound Addition*, in which other numbers besides ten, make units of higher orders. Thus, four units of the order of farthings, make one unit of the order of pence. Twelve units of the order of pence, make one of the shilling order. Twenty of the shilling order, make one of the pound order, &c.

The *fourth* is the *Addition* of *Vulgar Fractions*, in which *vulgar fractions* are added to each other. Thus $\frac{1}{5}$ $\frac{4}{5}$ and $\frac{2}{5}$ are added to each other.

What effect is produced by placing ciphers at the *right* of decimals? What effect have ciphers when placed at the *left* of decimals? Give examples of the abbreviations used in Arithmetic. What is Addition? Describe the four kinds of addition.

SIMPLE ADDITION.

If 8 units are added to 9 units, how many are there of the order of *tens?*

Write the 8 under the 9, and draw a line under. Place the *units* of the answer, under the figures added, and set the 1 ten before them.

If 13 apples are added to 25 apples, how many are there in the whole?

Write the units under units, and tens under tens. Add the units first, and place the answer under the unit column. Then add the tens in the same way.

Add 12 cents to 5 cents.
Add 13 apples to 14 apples.
Add 14 dollars to 19 dollars.
Add 5 and 2 and 12 together.
Add 13 and 12 and 14 together.

Let the pupil add small sums, which do not amount to ten of any order, till it can be done quickly, and with a full understanding of the process.

In the next process let the coins be used to illustrate.

If 25 cents be added to 16 cents, how many cents are there?

Let 2 dimes be laid on the table, and 5 cents placed at the right of them. Under the 2 dimes place 1 dime, and under the 5 cents place 6 cents. Let the pupil then add the 6 to the 5, and the answer will be 11 cents. Eleven cents are 1 dime and 1 cent. Let him leave 1 cent under the column of cents, and substitute 1 *dime* for the 10 cents. Let him place this dime with the 1 dime, and add the 2, and his answer will be 4 *dimes* 1 cent. Ask how many cents in 4 dimes 1 cent, and the answer will be 41 *cents*. Thus his answer will be *either* 4 dimes 1 cent, or 41 cents.

If the pupil thus sees the principle once illustrated, by a visible process, the method will be much more readily understood and remembered. Let the following sum also be done by the coins.

Add $1,36 to $2,97.

Add $2. 6d. 8 cts. to 3$. 8d. 9 cts.

Add 7 *E*. 2$. 5d. 6 cts. to 4 *E*. 8$. 6d. 4 cts.

Add 5d. 6 cts. 7 m. to 8d. 4 cts. 9m.

Add 4 *E*. 0$. 6d. 5 cts. to 5 *E*. 0$. 4d. 6 cts.

Let the teacher dictate such simple sums until the process of writing and adding is well understood, and can be done with *rapidity* and *accuracy*.

NOTE TO TEACHERS. It is very desirable that pupils should be required to *write* their figures with accuracy and neatness, and learn to place them in *straight* lines, both perpendicular and horizontal. Also that they learn to *add* by *calculation*, and not by *counting*, as young scholars are very apt to do. If a teacher will but *be thorough*, at the *commencement*, in these respects, much time and labor will be saved.

Mary has 4 apples, James 5, and Henry 7, how many have all together?

One boy has six marbles, another 4, and another 9, how many have all together?

A man gave 9 cents to one boy, 8 to another, and 11 to another, how many did he give to all?

10 and 11 and 9 are how many?

12 and 7 and 4 are how many?

4 and 5 and 7 are how many?

One man owns 6 horses, another 8, and another 9, how many have they all?

In a school, 10 study history, 11 geography, and 15 grammar, how many scholars in the whole?

One house has 10 windows, another 7, and another 12, how many are there in all?

James lent one boy 8 cents, another 6, and another 17, how many did he lend them all?

If a lady pays 7 dollars for a veil, 9 dollars for a dress, and 3 dollars for a necklace, what amount does she spend?

6 and 9 and 18 are how many?

10 and 5 and 7 are how many?

8 and 11 and 14 are how many?

Let the pupil be taught to add, using the *signs*. Thus the last sum. $8+11+14=33$.

RULE FOR SIMPLE ADDITION.

Place units of the same order in the same column, and draw a line under. Add each column separately, beginning at the right hand. Place the units of the amount, under the column to which they belong, and carry the tens to the next higher order.

Add 2694 and 3259 and 6438.

What is the rule for simple addition?

Placing units of the same order in the same column, they stand thus:

2694
3259
6438
12391

Let the pupil at first learn to add in this manner. 8 units added to 9, are 17, and 4 are 21 units, which is 1 of the *unit* order, to be written under that order, and 2 of the order of *tens*, to be carried to that order. 2 tens carried to 3 tens, are 5, and 5 are 10, and 9 are 19 tens; which is 9 of the order of *tens*, to be written under that order, and 1 of the order of *hundreds*, to be carried to that order. Thus through all the orders.

Add the following numbers, and let the pupil use the above method.

(1)	(2)	(3)	(4)
22321	23432	110331	222311
41332	42212	224212	131232
12123	13124	103123	101221
13220	21101	220320	234031
88996	99869	657986	688795

(5)	(6)	(7)	(8)
275496	456789	369543	4976432
8732	654321	695432	4976432
54976	456789	567897	6325498
843215	654321	432591	5192346
7621	543219	526387	8763945
49673	345678	489549	763497
1239713	3111117	3081399	30998150

(9)	(10)	(11)	(12)
30648	30430	764325	29367
46469	25895	70504	29367
74057	57644	98469	29367
63396	72919	57157	29367
55275	3647	46946	29367
90534	57246	3284	29367
8953	247781	363	176202
30142		1041048	
399474			

What is the method of adding the first example?

Exercises for placing figures in right periods and orders.

Let the pupil now learn to *place units of the same order in the same column*, by the following examples.

Let the teacher dictate the following. The pupils should be required *previously* to attempt writing them, while studying their lesson.

1.

One million, four hundred and sixty thousands, and two.

Twenty-four millions, six hundred and one.

Three hundred and sixty thousands, four hundred and six.

Ninety-four millions, five hundred and seventy-eight thousands, three hundred and forty-one.

Six millions, seven thousands, and forty-three.

2.

Two hundred and six thousands, five hundred and forty-two.

One million, one thousand, and one.

Nine hundred and ninety millions, nine hundred and ninety-nine.

Eighty-eight thousands, eight hundred and eighty-eight.

Ninety-nine millions, seven hundred and sixty-five thousands.

3.

Two hundred and six millions, five thousands, four hundred and one.

Fifty-six millions, four hundred thousands, five hundred and six.

Three billions, ninety-nine thousands, and four.

Five hundred millions, thirty thousands, four hundred and forty.

Seven millions, six hundred and fifty-four thousands, three hundred and seventeen.

4.

Four millions, four hundred and thirty-two thousands, one hundred and seventy-six.

Forty-nine thousands, and three.

Nineteen millions, seven hundred and sixty-five thousands, nine hundred and eighty-four.

Five hundred and ninety-one.

Seven hundred and sixty-three thousands, nine hundred and forty-three.

Ninety-nine millions, nine thousands and ninety.

5.

Four hundred and four.

Five millions, six hundred and forty-three thousands, two hundred and seventeen.

One million, and two. Nine thousands, and ninety-nine.

Four millions, five hundred and seventy-six thousand, three hundred and eighty-four.

Forty-four millions, three hundred and twenty-one thousands, seven hundred and four.

6.

One hundred millions, one thousand, and ten.

Nine billions, eight hundred thousands, nine hundred and forty.

Four hundred and eighty-eight millions, nine hundreds, and five thousands.

Eighty-eight millions, seven hundred and seventy-seven thousands, and nine.

Nine hundred and ninety-nine.

7.

Ninety-nine millions, eight thousands, and four.

Five hundred and eighty-seven millions, six hundred and forty-nine thousands.

Twenty-eight thousands, eight hundred and ninety-nine.

Four hundred thousands, eight hundred and seven.

One billion, fifty-nine millions, four thousand and eighty-seven.

8.

Seven hundred millions, ninety-nine thousands, and seventy-nine.

Fifty-five thousands, seven hundred and forty-four.

Nine millions, eight hundred thousands, eight hundreds.

Eight thousands, eight hundreds.

Seven billions, and seventeen.

METHOD OF PROVING ADDITION.

1. Commence at the top instead of the bottom of the several columns, and if the same answer is obtained, it may be considered as right.

2. Draw a line and cut off the upper figure of all the orders. Add the remainder which is not cut off. Then add the sum of this remainder to the figures cut off, and if the answer is the same as the first answer, it may be considered as right.

SUBTRACTION.

There are four kinds of Subtraction.

The *first* is *Simple Subtraction,* in which the minuend and subtrahend are whole numbers, and ten units of one order, make one unit of the next higher order.

The *second* is *Decimal Subtraction,* in which the minuend and subtrahend are Decimals.

The *third* is *Compound Subtraction,* in which other numbers besides ten, make units of a higher order.

The *fourth* is *Subtraction of Vulgar Fractions,* in which the minuend and subtrahend are vulgar fractions.

SIMPLE SUBTRACTION.

If 8 cents are taken from 12 cents, what will remain?

If 9 apples are taken from 14 apples, how many will remain?

If 12 guineas are taken from 20 guineas, how many will remain?

If from 18 books, 12 be taken, how many will remain?

Let the following examples be illustrated by the coin of the United States.

If $2, 5 d. 6 cts. be taken from $3, 6 d. 7 cts., how much will remain? Which is the subtrahend, and which the minuend?

Place $3, 6 d. 7 cts. on a table, side by side, and let the pupil take the amount of the subtrahend from them.

Subtract $3, 4 d. 5 cts. from $6, 7 d. 7 cts.

Subtract 3 d. 4 cts. 2 m. from 5 d. 6 cts. 8 m.

Subtract 8 d. 7 cts. 5 m. from 9 d. 9 cts. 9 m.

Let the teacher place on the table the coins, thus:

$3, 4 d. 6 cts.

Under this, place for the subtrahend, the following, so that the coins shall stand under others of the same order.*

$2, 2 d. 4 cts.

What is the remainder, when the value expressed by the subtrahend, is taken from the minuend?

Now if 10 cents be added to the 6 cents of the minuend, and

* The pupil must understand that the subtrahend shows how many of the same kinds of coin, are to be taken from the minuend.

What are the four kinds of subtraction? Describe them.

1 dime be added to the 2 dimes of the subtrahend, will there be any difference in the answer? Let the pupil try it and ascertain.

If 10 dimes be added to the 4 dimes of the minuend, and 1 dollar be added to the 2 dollars of the subtrahend, will there be any difference in the answer?

Let this process be continued until every member of the class fully understands it, and then let them commit to memory this principle.

If an equal amount be added to the Minuend and the Subtrahend the Remainder is unaltered.

Let the following coins be placed as minuend and subtrahend.

$	d.	cts.	
2	1	3	Minuend.
1	4	5	Subtrahend.

Which is the largest sum, *taken as a whole*, the minuend or subtrahend?

If each order is taken *separately*, in which orders is the minuend the largest, and in which the smallest?

Can you take 5 cents from 3 cents?

If you add 10 cents to the 3 cents, you can subtract 5 from it, but what must be done to prevent the Remainder from being altered?

	$	d.	cts.	m.
From	4	3	2	4
Subtract	1	4	5	6

In which orders are the numbers of the subtrahend larger than those of the minuend?

Can 6 mills be taken from 4 mills?

What can you do in this case?

If 10 mills be added to the 4 mills of the minuend, why must 1 cent be added to the 5 cents of the subtrahend?

From 6432, subtract 3256.

Can 6 units be taken from 2 units?

What must be done in this case?

RULE FOR SIMPLE SUBTRACTION.

Write the subtrahend under the minuend, placing units of the same order under each other, and draw a line under. Subtract each order of the subtrahend, from the same order of the minuend,

---

What is the principle by which the process of subtraction is performed?

and set the remainder under. If any order of the subtrahend is greater than that of the minuend, add ten units to the minuend, and one unit to the next higher order of the subtrahend. Then proceed as before.

EXAMPLE.

Subtract 4356
From 2187
2169

Let the pupil subtract thus:

Seven units cannot be taken from 6; therefore add 10 to the minuend, which makes 16. 7 from 16 leaves 9. As 10 units have been added to the minuend, the same amount must be added to the subtrahend. 1 of the order of tens is the same amount as 10 units, we therefore add 1 to 8 tens, making it 9 tens. We cannot subtract 9 tens from 5 tens, we therefore add 10 to the minuend, which makes 15. 9 tens from 15 leaves 6 tens. As 10 tens have been added to the minuend, the same amount must be added to the subtrahend—1 of the order of hundreds is the same amount as 10 tens; we therefore add 1 to 1 hundred, which makes 2 hundred. This subtracted from 3 hundred leaves 1 hundred.

Thus through all the orders.

Mode of Proof.

A sum in Subtraction is *proved* to be right, by adding the *remainder* to the *subtrahend*; and if the sum is the same as the *minuend*, the answer may be considered as right.

Let the following sums be explained as above.

Subtract	34695	from	56943
"	653215	"	956432
"	500032	"	867200
"	6291540	"	8732418
"	354965	"	5360025
"	7985430	"	989763
"	3542685	"	6542169
"	5321543	"	7954324
"	1223345	"	8500642
"	1549768	"	3895463
"	3543257	"	6385241
"	2006935	"	5000623

What is the rule for simple subtraction? What is a mode of proof?

The pupil should learn to subtract by the use of the *signs*, thus:

Subtract 5 from 7. Ans. 7—5=2.

Subtract 8 from 11. Ans. 11—8=3.

Subtract the following numbers in the same way. 8 from 17. 9 from 14. 6 from 20. 40 from 85. 800 from 950. 1000 from 2744. 85 from 760. 95 from 700. 440 from 763.

MULTIPLICATION.

Multiplication is repeating a number, as often as there are units in another number.

The number to be repeated is called the *multiplicand.*

The figure expressing the number of times the multiplicand is to be repeated, is called the *multiplier.*

The *answer* is called the *product*, because it is the sum *produced* by multiplication.

The multiplier and multiplicand are called the *factors*, from the Latin word *factum*, (*made*,) because they are the numbers by which the product is made.

There are four processes of multiplication.

The first is *Simple Multiplication*, where the factors are whole numbers, and ten units of one order make one unit of the next higher order.

The second is *Decimal Multiplication*, where one, or both the factors are decimals.

The third is *Compound Multiplication*, where the multiplicand consists of orders, in which other numbers besides *ten*, make units of a higher order.

The fourth is the multiplication of *vulgar fractions*, where one, or both the factors, are vulgar fractions.

SIMPLE MULTIPLICATION.

A boy gives 8 apples to each of 7 companions, how many does he give to them all?

A man travels 7 miles an hour, how far will he travel in 9 hours?

If one pound of raisins cost 11 cents, how much will 6 pounds cost?

Describe the four different kinds of multiplication.

One boy has 7 cents, and another twelve times as many, how many has the last?

At six cents apiece, how much will 9 lemons cost?

At 12 cents a dozen, how much will 8 dozen marbles cost?

One pound of sugar cost 6 cents, how much will 5 pounds cost? 8 pounds? 11 pounds? 12 pounds?

Multiplication has been defined as *repeating*, or *taking* one number as often as there are units in another number. Let this process be illustrated by the coins; thus,

$		*d.*		*cts.*
2	"	4	"	3

Let the multiplier be 2.

Now the pupil is to *take* 3 cents, as often as there are units in 2, and give the answer. Then he is to take 4 dimes as often as there are units in 2, and then 2 dollars in like manner.

Let the following sum be done by the coins.

	$		*d.*		*cts.*
	2	"	4	"	4
Multiplied by					3

When the pupil has taken 4 cents three times, he will have 12 cents. Let a dime be substituted for *ten* of these cents, to be carried to the next product, and there remain two cents, to be placed in the order of cents. Then let 4 dimes be taken 3 times, which make 12, and the one dime of the other product is added, making 13 dimes. Let a dollar be substituted for *ten* of the dimes, and carried to the next product, and three dimes will remain to be placed in the order of dimes. Two dollars taken three times, will make 6 dollars, and adding the one dollar of the other product, the amount is 7 dollars, to be placed in the order of dollars.

The pupil should practice in this way until the principle is fully understood.

RULE FOR MULTIPLYING, WHEN THE MULTIPLICAND HAS SEVERAL ORDERS, AND THE MULTIPLIER DOES NOT EXCEED TWELVE.

Place the multiplier below the multiplicand. Beginning at the right, multiply each order of the multiplicand, by the multiplier.

What is the rule for Simple Multiplication, when the multiplier does not exceed 12?

Place the units of the product, under the order multiplied, and carry the tens to the next product. Write the whole of the last product.

Let the pupils at first be exercised thus:—

EXAMPLE.

```
 249
   8
----
1992
```

Eight times 9 units are 72 units; which is 2 units to be written under that order, and 7 tens to be carried to the next product. Eight times 4 tens, are 32 tens, and the 7 tens carried, make 39 tens, which is 9 of the order of tens, to be written under that order, and 3 hundreds to be carried to the next product. Eight times 2 hundreds, are 16 hundreds, and the 3 hundreds carried, make 19 hundreds, which are written down.

EXAMPLES.

Multiply	348	by	4.	Multiply	2469	by	6.
"	728	"	5.	"	6923	"	7.
"	4693	"	6.	"	4593	"	8.
"	2914	"	7.	"	12468	"	9.
"	3463	"	8.	"	42469	"	10.
"	6798	"	9.	"	53273	"	5.
"	5124	"	10.	"	65492	"	8
"	8763	"	11.				

When the *multiplier* consists of *several orders*, another method is adopted. For example,

Multiply 324 by 67.

The 324 is first to be multiplied by the 7 units, according to the former rule, and the figures stand thus,

```
 324
  67
----
2268
```

The 324 is now to be multiplied by the 6; what is the number represented by the 6? Ans. 60 or 6 tens.

If 4 is multiplied by 6 tens, the answer is 24 *tens*, or 240. The 4 is to be written in the order of tens, under the 6, [The cipher is omitted because, by setting it under the 4, we can know what order it is without a cipher,] and the 2 (which is 200) is to be carried to the next product.

```
  324
   67
-----
 2268
1944
-----
21708 Ans.
```

In the sum above, of what order is each of the figures? What is the product of 4 units multiplied by 6 tens? Why is the cipher omitted?

The 2 *tens*, or (20), are next multiplied by the 6 tens, (or 60) and the answer is 12 *hundreds*, (1200) and the 2 hundreds to be carried to it, make 1400. The 4 is written in that order, and the 1 carried to the next product. Next the 3 hundreds are multiplied by the 6 tens, and the answer is 18 thousands, (18000) and the one to be carried to it, make 19 thousands, which are placed in their orders. Then the two products are *added* together, and the answer is obtained.

Let the pupil answer the following questions on the above sum.

What number does the 6 of the multiplier, represent? What number does the 2 represent? If they are multiplied together, as if they were *units*, what is the product? How many ciphers must be added, to express the true value of 2 tens, multiplied by 6 tens? How many figures are at the *right hand* of both the factors, 2 tens and 6 tens? Is the number of *ciphers* added, the same as the number of *figures* at the right hand of both the factors?

What is the answer if the 3 hundreds be multiplied by 6 tens, as if they were *units?* How many ciphers must be added, to make the product express the true value? Does the number of ciphers added, correspond to the number of figures, at the *right* of both factors?

By answering the above questions, the pupil will understand the following principle.

Figures of any order may be multiplied together like units, and the true value is found, by annexing as many ciphers as there are figures at the right of both the factors.

Let the following questions be answered.

Multiplicand 869
Multiplier 237

What number is represented by 6? by 3?

If the 6 is multiplied by the 3, what is the answer, if the factors are considered as units? What is the true answer?

If the 8 is multiplied by 3, what is the answer if they are considered as units? What is the true answer?

What number is represented by 2? by 8? If the 2 is multiplied by 8, what is the answer if they are considered as units? What is the true answer?

What is the product of 2 tens multiplied by 6 tens, and how is it set down? How can the *true value* of any orders that are multiplied together be found? Give examples.

Let the pupil now learn to multiply the above sum, and place the figures in the orders to which they belong; thus,

```
   869 Multiplicand.
   237 Multiplier.
  ----
  6083
 2607
1738
------
205952 Answer.
```

The mutliplicand is first multiplied by the 7 of the multiplier, and the product is 6083.

Then the 3 *tens* (or 30) are multiplied into the 9 units, and the answer is 270; which is 7 tens to be set in the order of tens, and 2 hundreds to be carried to the next product.* Then the 6 tens (or 60) are multiplied by 3 tens, and the product is 1800, and the 2 that were to be carried make 2000; which is 2 of the order of thousands to be carried to the next product, and 0 to be set in the order of hundreds. Then the 8 hundreds are multiplied by 3 tens, and the answer is 24000, and the 2 to be carried make 26000; which is 6 to be set in the order of thousands, and 2 in the order of tens of thousands.

Next take the 2 hundred as multiplier, and multiply 9 units by it, and the answer is 1800; which is 8 to be set in the order of hundreds, and 1 to be carried to the next product.

Proceed thus, till all the orders have been multiplied by the 2 hundred. Then add the several products and the answer is obtained.

RULE FOR SIMPLE MULTIPLICATION, WHEN THE MULTIPLIER HAS SEVERAL ORDERS.

Place the multiplier below the multiplicand, so that units of the same order may stand in the same column. Multiply by each order of the multiplier. Write the units of each product, in the order to which they belong, and carry the tens to the next product. Add the products of the several orders, and the sum is the answer.

* The cipher is omitted because, as the figure is set under the 8, we can tell to what order it belongs without the cipher.

What is the product of 9 units multiplied by 3 tens, in the above sum? In writing it why is the cipher omitted? What is the product of 6 tens multiplied by 3 tens, and how is it to be written? What is the product of the other orders, and how are they written? What is the rule for Simple Multiplication, when the multiplier has several orders?

EXAMPLE.

```
   826
   234
  ----
  3304
 2478
1652
------
193284
```

Multiply by the 4 units according to the other rule.

Then multiply each order of the multiplicand by the 3 tens (or 30) thus: 6 units multiplied by 3 tens are 18 tens, which is 8 tens to be written in that order, and 1 of the order of hundreds to be carried to the next product. 2 tens, (or 20) multiplied by 3 tens (or 30) are 600, and the 100 carried, makes 700, which is 7 to be written in the order of hundreds. 8 hundreds multiplied by 3 tens, (or 30) is 24000; which is 4, to be written in the order of thousands, and 2 tens of thousands to be set in that order.

Lastly, multiply each order of the multiplicand by the 2 hundreds. 6 units multiplied by 2 hundreds, are 12 hundreds, which is 2 hundred to be written in that order, and 1 thousand to be carried to the next product. 2 tens (or 20) multiplied by 2 hundreds, are 4000, and the 1000 carried makes 5000, which is 5 to be placed in the order of thousands. 8 hundreds multiplied by 2 hundreds, are 160,000, which is 6 tens of thousands, to be written in that order, and 1 hundred of thousands, to be written in the order of hundreds of thousands.

Add all the orders of the products, by the rule of common addition, and the sum is the answer.

EXAMPLES.

Multiply	256	by	26	Multiply	4567	by	234
"	3639	"	329	"	4654	"	496
"	4638	"	462	"	6789	"	596
"	5943	"	567	"	5432	"	281
"	2345	"	234	"	4568	"	362
"	7892	"	456	"	8382	"	945

If five stands alone (5) of what order is it? If a cipher is affixed, of what order is it? How much larger is the sum, than it was before? By what number was it multiplied when the cipher was added?

In the above sum, what are the products of the several orders, and how are they written?

If two ciphers are added to the 5, in what order will it stand? How much larger is the sum than it was before? By what number was it multiplied when the ciphers were added?

If three ciphers are added to 5, in what order will it stand? How much larger is the sum than it was before? By what number was it multiplied when the ciphers were added?

If you wish to multiply 5, by 10, what is the shortest way? If you wish to multiply 5, by 100, what is the shortest way? If you wish to multiply 5, by 1000, what is the shortest way?

If you wish to multiply 50, by 2, how would you do it?

Would it make any difference if you should multiply the 5 first, and then affix a cipher to the answer?

If you are to multiply 5000, by 2, can you begin by multiplying the 5 first?

If you are to multiply 35000, by 2, can you multiply the 5 first, and then the 3, and afterwards affix the three ciphers?

If you are to multiply 20 by 30, can it be done by multiplying the 3 and 2 together, and then affixing 2 ciphers to the product?

Multiply 200 by 20 in the same way.

RULE FOR MULTIPLYING WHEN THE FACTORS ARE TERMINATED BY CIPHERS.

Multiply the significant figures together, and to their product annex as many ciphers as terminate both the factors.

Note.—All figures are called *significant*, except ciphers.

Multiply	30	by	20	Multiply	2400	by	2000
"	3000	"	9	"	160	"	4200
"	200	"	6	"	400	"	60
"	2000	"	40	"	96	"	30
"	100	"	100	"	4400	"	90

When any number is made by multiplying two numbers together, it is called a *composite number*.

Thus 12 is a composite number, because it is made by multiplying 3 and 4 together.

Is 18 a composite number? What two numbers multiplied together make 18?

Is 14 a composite number? Is 13 a composite number? Is 9 a composite number?

What is the rule for multiplying when both factors are terminated by ciphers? What is a composite number?

If 12 is multiplied by 8, what is the product? What are the factors which compose 8?

If you multiply 12 by one of these numbers, and the product by the other, will the answer be the same as if you multiply 12 by 8? Let the pupil *try* and *see*.

What are the numbers that compose 18?

Multiply 123 by 18. Multiply it by one of the numbers that compose 18, and the product by the other number, and what is the result?

RULE FOR MULTIPLYING, WHEN THE MULTIPLIER EXCEEDS 12, AND IS A COMPOSITE NUMBER.

Resolve the multiplier into the factors which compose it, and multiply the multiplicand by one, and the product by the other.

Let the following sums be done by the above rule.

Multiply	33	by	20	Multiply	6543	by	40
"	268	"	49	"	587	"	16
"	329	"	54	"	6543	"	24
"	426	"	32	"	521	"	27
"	2345	"	96	"	72	"	30
"	7654	"	64	"	793	"	36

In multiplication it makes no difference in the product, which of the factors is used for multiplier or multiplicand; for 3 times 4, and 4 times 3, give the same product, and thus with all other factors. It is in most cases most convenient to place the *largest* number as multiplicand.

DIVISION.

Division is finding how often one number is contained in another, and thus finding what part of one number is another number.

The number to be divided is called the *Dividend*.

The number by which we divide is called the *Divisor*.

The answer is called the *Quotient*.

What is left over, after division, is called the *Remainder*.

There are four kinds of division.

What is the rule for multiplying when the multiplier exceeds 12, and is a composite number? Does it make any difference which factor is multiplier? Which is most convenient for multiplier? What is Division? What are the Divisor and Dividend? Quotient? Remainder?

The *first* is *Simple Division*, in which both the dividend and divisor are whole numbers, and ten units of one order, make one unit of the next higher order.

The *second* is *Compound Division*, in which other numbers besides ten, make units of higher orders.

The *third* is *Division of Vulgar Fractions*, in which the dividend or divisor (or both) are Vulgar Fractions.

The *fourth* is *Decimal Division*, in which the dividend, or divisor, (or both) are Decimal Fractions.

SIMPLE DIVISION.

How many 9 cents are there in 63 cents?

What part of 63 cents is 9 cents?

How many times is 8 contained in 56?

If 8 is contained 7 times in 56, what part of 56 is 8?

If 56 is divided by 8, how much smaller is the quotient than the dividend?

How many 7 dollars are there in 42 dollars?

What part of 42 dollars is 7 dollars?

How many times is 6 contained in 66?

If 6 is contained 11 times in 66, what part of 66 is 6?

If 66 is divided by 6, how much smaller is the quotient than the dividend?

There are many numbers which cannot be divided into equal parts, without making a fraction. For example, if we wish to divide 7 apples into two equal portions, we should have for answer 3 apples and ½ of an apple.

If we had 13 apples, and wished to give a third of them to each of 3 friends, we should divide the 13 by 3, and the answer would be 4, and 1 left over. That is, we could give 4 apples to each of the 3 friends, and *one* would be left to divide among them. This divided by 3, (or into 3 equal parts) would give a *third* to each one. 13 then, divided by 3, gives 4 and ⅓ as answer.

If you are to divide 7 apples equally among 3 persons, how many *whole* apples would you give to each, and what would remain to be divided?

If you had 14 oranges, and wished to divide them equally

What are the four kinds of Division?

among 6 persons, how many *whole* oranges would you give each?

How would you divide the two that remained?

Ans. Divide each into 6 equal parts, and give *one* of the parts of *each* orange to the 6 persons. Each person would then have 2 oranges and $\frac{2}{6}$.

If you have two apples, each cut into 12 parts, and take 4 of these parts from each apple, how much do you take?

Ans. $\frac{8}{12}$. For $\frac{4}{12}$ from each apple makes $\frac{8}{12}$ in the whole.

If we take 9 twelfths from each of the two divided apples, we shall have $\frac{18}{12}$ in the whole.

Now this is not $\frac{18}{12}$ of *one* apple, for nothing has more than 12 twelfths. Whenever therefore we find an *improper* fraction, we know that *more* than *one unit* has been divided.

What part of 13 apples is 3 apples and $\frac{1}{4}$ of an apple?

Ans. It is a *fourth* of 13, because 4 times 3 and $\frac{1}{4}$ make 13.

What part of 5 is 1? is 2? is 3? is 4? is 6?

In the last question we reason thus · if 1 is *one fifth* of 5, 6 must be 6 times as much, or $\frac{6}{5}$ of 5.

What part of 8 is 1? is 4? is 7? is 9?

What part of 15 is 1? is 2? is 3? is 14? is 19?

What part of 10 is 1? is 2? is 5? is 9? is 11? is 20?

What is a sixth of 19? What is a fourth of 21?

What is an eighth of 26?

If you had 19 pears, and divided them equally among 6 persons, how much would you give to each?

What part of 19 is 3 and $\frac{1}{6}$?

Ans. As there is 6 times 3 and $\frac{1}{6}$ in 19, it is $\frac{1}{6}$ of 19.

When one number is placed over another, it signifies that the upper number is divided by the lower.

Thus, $\frac{3}{4}$ signifies that the 3 is divided by 4. For a fourth of three things is 3 *fourths*, and $\frac{3}{4}$ signifies either 3 fourths of *one* thing, or *a fourth* of 3 things.

If you wish to divide 3 dollars into 5 equal parts, what would it be necessary to do, before you *could* divide them?

Ans. Change them to dimes.

What would be the answer?

If you wished to divide 4 dimes into 10 equal parts, what would it be necessary to do before you could divide them?

What would be the answer?

(Let this be shown by the coins.)

How can 3 dollars be divided so as to give ten of the class, each an equal part?

Ans. Change the dollars to dimes, and then dividing them into ten equal parts, there will be 3 dimes for each of the ten.

Divide $ 1,2 so as to give six scholars, each an equal part.

Divide $ 2,4 so as to give 8 scholars, each an equal part.

Divide 1 dime equally between two scholars.

Divide 1 dime 5 cents equally between 3 scholars.

If 1 dime 8 cents are divided by 6, what is the answer?

If 3 dimes 9 cents are divided by 6, what is the quotient, and what the remainder?

If 5 dimes 6 cents are divided by 7, what are the quotient and remainder?

If 4 dimes 7 cents are divided by 6, what are the quotient and remainder?

In the above sums, it will be seen that *when one order of the dividend will not contain the divisor once, it is reduced, and added to the next lower order, and then divided.*

Thus when 4 dimes, 6 cents were to be divided by 6, the 4 dimes were changed to cents, and added to the 6 cents, and then divided.

It will also be seen, that *the quotient and the remainder are always of the same order as the dividend.*

Thus if 4 dimes 7 cents are divided by 6, the 4 dimes are reduced, and added to the cents, and the quotient is 7 *cents*, and the *remainder* is 5 *cents*.

Thus, also, if 17 *thousands* are divided by 5, the quotient is 3, and 2 remainder. The 3 is 3 *thousands*, and the 2, is 2 *thousands*.

If the order of the dividend were *millions*, the quotient and remainder would also be millions.

If the order were *tens* the quotient and remainder would also be tens.

If we divide 8 *tens* by 3, the quotient is 2 *tens*, and the remainder 2 *tens*.

When the dividend has several orders, we divide each order *separately*, beginning with the highest orders. This is called *Short Division*.

What is the method of dividing when one order of the dividend will not contain the divisor once? Of what order are the quotient and remainder?

If there is any remainder, after the division of each order, it is changed to the next lower order, added to it, and then divided.

For example. Let 9358 be divided by 4.

We first divide the 9 thousands by 4, add the remainder to the 3 hundreds and divide that. Then divide the tens and units. Place them thus:

$$\begin{array}{r} 4)9358 \\ \hline 2339\frac{2}{4} \end{array}$$

The 9 *thousands* is first divided. In nine *units* there would be 2 fours, and 1 remainder. But as this is 9 *thousands*, the quotient and remainder must be the same order as the dividend, and the 2, is 2 *thousand* fours, and is set under the 9 in the thousands order. The remainder also is 1 *thousand*, and is changed to hundreds and added to the 3, making it 13 hundred. This is then divided by 4. The quotient is 3 *hundreds*, which is put under that order, and the 1 hundred that remains, is changed to tens and added to the 5 tens, making 15 tens. This is divided by 4, and the quotient is 3 *tens*, which is set in that order. 3 tens remain, which, changed to units and added to the 8, make 38 units. This is divided by 4, and the quotient is 9 units, which is put in that order. 2 units remain, which are divided by the 4 thus $\frac{2}{4}$.

9358, then, contains 4, 2 thousands of times, 3 hundreds of times, 3 tens of times, and 9 units of times. The 2 left over, is $\frac{2}{4}$ of another time.

Let the pupil, in performing each operation on the slate, explain it thus:

NOTE TO TEACHERS.—Let such questions as those below be asked on several sums, till the pupil fully understands them.

$$\begin{array}{r} 7)2496 \\ \hline 356\frac{4}{7} \end{array}$$

7 is contained in 24 units 3 times, in 24 hundreds, 3 *hundred* times, which are set in the order of hundreds. 3 *hundred* are left over, which, changed and added to the 9 tens, make 39 tens.

7 is contained in 39 tens, 5 *tens* of times, which are set in the

---

In the first example, what is divided first? Of what order is the first quotient figure, and why? What is done with the remainder? Explain the remainder of the sum in the same way.

order of tens. 4 *tens* are left over, which, changed and added to 6, make 46 units.

Divide 46 units by 7, and the answer is 6 *units*, which are set in that order, and 4 remain, which have the 7 set under them, to show that they are divided by 7.

RULE FOR SHORT DIVISION.

Divide the highest order, and set the quotient under it. If any remains, reduce and add it to the next lower order, and divide as before. If the number in any order, is less than the divisor, place a cipher under it in the quotient; then reduce and add it to the next lower order, and divide as before. If any remains when the lowest order is divided, place the divisor under it as a fraction.

EXAMPLES.

Divide	3694	by	3	Divide	3456	by	3
"	4329	"	4	"	7892	"	4
"	6548	"	5	"	3456	"	5
"	3621	"	6	"	7892	"	6
"	4638	"	7	"	1234	"	7
"	29639	"	8	"	5678	"	8
"	36964	"	9	"	91234	"	9
"	24697	"	10	"	56789	"	10
"	36941	"	11	"	12345	"	11
"	1263	"	12	"	67891	"	12

When *both the divisor and dividend*, have *several* orders, another method is taken, called *Long Division*. Let 6492 be divided by 15. In performing the operation described below, we set the figures thus:

Dividend.

Divisor 15)6492(432$\frac{12}{15}$ Quotient.
60
―――
49
45
―――
42
30
―――
12

We first take as many of the highest orders as would *if units*, contain the divisor *once*, and *not more* than 9 times. In this case we take 64 *hundreds*. Now we cannot very easily find *exactly* *how many times* the 15 is contained in 64 hundreds. But we can find how many *hundreds of times* it is contained thus. As 15 would be contained 4 *units* of times, in 64 *units*, it is contained 4 *hundreds* of times, in 64 *hundreds*. Which 400 is to be set in the quotient, (omitting the ciphers.)

As we have found that the dividend contains 15, 4 *hundreds* of times, we subtract 4 hundred times 15 from the dividend, to find how often 15 is contained in *what remains*. 400 times 15 is 60 hundreds (6000) which, subtracted from the 64 hundreds, leaves 4 hundreds.

This 4 hundreds changed to *tens*, and the 9 *tens of the dividend* put with it, make 49 *tens*. We now find how many *tens of times* the 15 is contained in the 49 tens, thus: as 15 would be contained 3 *units* of times in 49 *units*, it is contained 3 *tens* of times in 49 *tens*, which 3 tens is set in the quotient. We now subtract 3 tens of 15 (or 45 tens) from the 49 tens, and 4 *tens* remain. These are changed to *units* and have the 2 *units of the dividend* put with them, making 42 units. 15 is contained in 42 units 2 *units* of times, which is set in the quotient. Twice 15 from 42 units, leave 12, which is $\frac{12}{15}$ of another 15. The 15 then, is contained in the dividend 4 *hundreds* of times, 3 *tens* of times, 2 *units* of times, and $\frac{12}{15}$ of another time, or 432 times, and $\frac{12}{15}$ of another time.

Again, divide 6998 by 24.

To do it we *first* find how many *hundreds* of times the dividend contains the divisor, and subtract these hundreds; *second*, how many *tens* of times, and subtract these tens; *third*, how many *units* of times, and subtract these units; and *fourth*, what remains has the divisor set under it.

```
24)6998(291 14/24
   48
   ---
   219
   216
   ---
    38
    24
    ---
    14
```

Let the pupil, in doing sums, explain them as below.

24 is contained in 69 *units*, 2 times; in 69 *hundreds*, 2 hun-

---

What is the rule for Short Division? When is Long Division performed? How many of the highest orders are first taken? Do we find *exactly* how many times the divisor is contained? What do we find, and how do we reason in order to find it? What is the first quotient, and what is omitted in setting it down? After we have found how many hundred times the divisor is contained, what is done next, and for what purpose? What is done with the 4 hundred that remain?

dred times. 2 hundred times 24 is 48 hundred, which subtracted from 69 leaves 21 hundred.

21 *hundreds* are 210 *tens*, and the 9 tens of the dividend brought down, make 219 tens.

24 is contained in 219, 9 times; in 219 *tens*, 9 *tens* of times. 24 multiplied by 9 tens, is 216 tens, which subtracted from 219 tens leaves 3 tens.

3 tens are 30 units, and the 8 units of the dividend brought down make 38 units. 24 is contained in 38 units once, and 14 over, which is $\frac{14}{24}$ of another time.

The dividend then contains the divisor 2 hundreds of times, 9 tens of times, 1 unit of times, and $\frac{14}{24}$ of another time, or 291 times and $\frac{14}{24}$ of another time.

Thus it appears, that in *Long Division*, each quotient figure, when set down, does not show the *exact* number of times the divisor is contained in *the order which is divided;* but it shows, that the divisor is contained *so many times* as the quotient figure expresses, and then, a process follows for discovering *how many more times* it is contained.

Let the pupil do the following sums, and explain them as above, until perfectly familiar with the mode.

Divide	2479	by	14	Divide	3568	by	16
"	1954	"	18	"	5896	"	23
"	36964	"	17	"	38907	"	21
"	29006	"	28	"	46032	"	36

RULE FOR LONG DIVISION.

Place the divisor at the left of the dividend, and draw a line between. Take as many of the highest orders as would, if units, contain the divisor once, and not more than 9 times. Divide the orders so taken, as if they were units. Place the quotient figure at the right of the dividend, and draw a line between. Multiply the quotient and the divisor together, and subtract them from the part of the dividend already divided. To the remainder, add as many of the next undivided orders of the dividend as would enable

Explain the remainder of the process. In the second sum what is done first? second? third? fourth? Explain the whole process. In Long Division what does each quotient figure *not* show? What *does* it show? What process follows? What is the rule for Long Division?

it, if units, to contain the divisor once, and not more than 9 times, and then divide as before.

If it is needful to add more than one order of the dividend to any remainder, (to enable it to contain the divisor) put one cipher in the quotient for every additional order. If any remains after dividing the unit order, put the divisor under it for a fraction.

EXAMPLES,

Divide	2649	by	12	Divide	3294	by	14
"	2468	"	15	"	64329	"	16
"	1234	"	17	"	5678	"	18
"	56789	"	19	"	8234	"	36
"	35673	"	59	"	76542	"	41
"	45678	"	256	"	96743	"	348
"	912345	"	481	"	59624	"	562
"	678122	"	984	"	23864	"	541
"	34568	"	639	"	35469	"	856
"	543219	"	656	"	1459862	"	942
"	678912	"	9481	"	724368	"	2586
"	9876533	"	6002	"	159864	"	2851

EXAMPLES FOR MENTAL EXERCISES.

1. Bought 12 pounds of raisins for 3 shillings a pound, how many dollars did they cost?

State the process thus. If *one* pound cost 3 shillings, 12 pounds cost 12 times as much, or 36 shillings. As there are 6 shillings in a dollar, they cost as many dollars as there are sixes in 36.

Let the following sums be stated in the same manner.

2. Bought 5 bushels of peaches at 4 shillings a bushel, how many dollars did they cost?

3. How many peaches at 4 cents each must you give for 9 oranges at 5 cents apiece?

State the last sum thus. If one orange cost 5 cents, 9 cost 9 times as much, or 45 cents. As each peach is worth 4 cents, you must give as many peaches as there are *fours* in 45.

4. If you buy 10 yards of cotton, at 5 shillings a yard, and pay for it with butter at 2 shillings a pound, how many pounds will pay for it?

5. How many apples at 4 cents each, must you give for 3 pine apples at 12 cents each?

6. If you buy 48 bushels of coal for 12 cents per bushel, and

pay for it with cheese at 10 cents per lb. how many pounds do you give?

7. How much rye at 5 shillings a bushel must you give for 12 bushels of wheat at 8 shillings a bushel?

8. How much cloth worth 9 shillings a yard must you give for a firkin of butter worth 12 dollars?

(Change the dollars to shillings.)

9. How many dozen of eggs at 9 cents per dozen must be given, for 3 yards of cotton worth 20 cents per yard?

10. If you have 8 pine apples worth 9 cents each, and your companion has 9 quarts of strawberries worth 8 cents a quart, which he gives to buy the same worth of pine apples, how many pine apples must you give him?

REDUCTION.

Reduction is changing units of one order, to units of another order.

Reduction Ascending, is changing units of a *lower* to a *higher* order.

Reduction Descending, is changing units of a *higher* to a *lower* order.

EXAMPLES FOR MENTAL EXERCISE.

In 4 gallons how many quarts?

NOTE.—Let each sum be *stated* thus. *One* gallon contains *four* quarts, and *four* gallons four times as much. 4 times 4 is 16.

In 4 gallons how many pints?

In 8 yds. 3 qrs. how many quarters?

In 8 feet how many inches?

In 4 bushels how many quarts?

In 5 hours how many minutes?

Are the above sums in Reduction Ascending or Descending?

In 32 quarts how many gallons?

Let such sums be *stated* thus. *One* gallon contains 4 quarts. In 32 quarts therefore, there are as many gallons as there are 4's in 32.

In 42 pints how many gallons?

In 49 quarters how many yards?

What is reduction? What is reduction ascending? descending?

In 56 nails, how many quarters and how many yards?
In 64 inches how many feet?
In 36 barley corns how many inches?
In 96 quarts how many bushels?
In 120 minutes how many hours?
In 48 feet how many yards?
In 94 inches how many feet?
In 3 yards how many inches?
In 4 gallons how many pints?
In 32 quarts how many gallons?
In 80 penny weights how many ounces?
In 24 ounces how many penny weights?
In 8 pounds how many shillings?
In 40 shillings how many pence?
In £2, 9s. 6d. 3qrs. how many farthings?
In doing this sum we proceed in the following manner:

```
 £     s.     d.    qr.
 2  "  9  "   6  "  3
 20 "
 ----
 49 shillings.
 12
 ----
 594 pence.
   4
 ----
 2379 farthings.
```

We first change the pounds to *shillings*, by multiplying by 20, and add the 9 shillings to them, making 49 shillings.

We then change the 49 shillings to *pence*, by multiplying by 12, and add the 6 pence to them, making 594 pence.

We then change the 594 pence to *farthings*, by multiplying by 4, and add the 3 qrs. and thus we obtain the answer, 2379 qrs.

This is Reduction Descending, because we have changed units *of a higher order to those of a lower*.

Why did we multiply by 20, 12, and 4?

Let us now reverse the process, and change 2379 farthings to pounds. We proceed thus:

```
           £     s.    d.    qr.
 4)2379(2  "  9  "  6  "  3.
 12)594
  20)49
      2
```

We first change the 2379 farthings to *pence*, by dividing by 4, and the answer is 594 pence, and 3 farthings (or qr.) over, which is put in the quotient with qr. over it.

We then change the 594 pence to *shillings*, by dividing by 12, and the answer is 49 shillings, and six pence over, which is put in the quotient with d. written over.

We next change the 49 shillings to *pounds*, by dividing by 20, and find there is £2 and 9s. over, which are both put in the quotient with their signs written over them.

Why did we divide by 4, 12, and 20?

Let the following sums be performed and explained in the same way. Change 2486 farthings to pounds.
Change £2 18s. 4d. 2 qr. to farthings.
Change 241 shillings to pounds.
Change 249 pence to shillings and pounds.
Change £21 2s. to farthings.
Change 361 pounds to pence.
Change 35 shillings to pounds.

RULE FOR REDUCTION.

To reduce from a higher to a lower order.

Multiply the highest order by the number required of the next lower order, to make a unit of this order. Add the next lower order to this product, and multiply it by the number required of the next lower order, to make a unit of this order, adding as before. Thus through all the orders.

To reduce from a lower to a higher order.

Divide the amount given, by the number required to make a unit of the next higher order. Divide the answer in the same way, and continue thus till the answer is in units of the order demanded. The remainders are of the same order as the dividend, and are to be put as a part of the answer.

EXERCISES.

Bought a tankard of silver weighing 5lb. 8oz. for which I paid $1,12 an ounce, how much did it cost?

Reduce 2 lb. 8 oz. 11 pwt. to grains.

In 8 lb. 9 ℥. 4 ʒ. 2 ℈. 16 grs. how many grains?

In 11924 grains how many pounds?

What cost 4 cwt. 3 qrs. 17 lbs. of sugar, at 12½ cts per. lb.

What is the rule for reduction?

In 436 boxes of raisins, each containing 24 lbs. how many cwt?

In 63469542 drams, how many tons?

In 546 yards how many nails?

In 5486 nails how many yards?

In 118¾ yards, how many Ells Flemish?

How many barley corns will reach round the globe, it being 360 degrees?

How many miles in 836954621 barley corns?

In 18 acres, 3 roods, 12 rods, how many square feet?

How many square feet in 16 square miles?

In 9269546231 square feet how many square miles?

In 37 cords of wood how many solid feet?

In 20486 solid feet how many cords?

In 4 pipes of wine how many pints?

In 9120854 pints how many pipes?

In 464 bushels how many quarts?

In 964693 pints how many bushels?

COMPOUND ADDITION.

In order to understand the following sums, the pupil must commit to memory the tables inserted in the commencement of the book.

Sums for Mental Exercise.

If a man has 2 lbs. 10 oz. of beef, and buys 6 lbs. 8 oz. more, how much has he in the whole? First add the ounces. In 18 oz. how many pounds, and how many ounces over? Set down the ounces that are over, and add the pound to the other pounds, and what is the answer?

A boy has 3 yards 2 quarters of cloth, and buys 2 yards and 3 quarters more, how much has he in the whole?

One man buys 3 bushels and 2 pecks of grain, another buys 2 bushels and 3 pecks, how much do both together buy?

If you have 1 quart and 1 pint of milk, and buy 2 quarts and 1 pint more, how much will you have?

One rope is 3 feet, 7 inches long; another is 4 feet, 6 inches; how many feet are there in both together?

If 2 weeks 4 days be added to 1 week 5 days, how many *weeks* will there be in all?

If 6 pounds 9 oz. be added to 5 pounds 8 oz. how many *pounds* will there be in all?

If 3 bushels 2 pecks be added to 4 bushels 3 pecks, how many bushels will there be?

If 7 yards 2 quarters be added to 8 yards 3 quarters, how many yards will there be?

RULE FOR COMPOUND ADDITION.

Place units of the same order in the same column. Find the sum of each order. Find how many units of the next higher order are contained in the sum, and carry them to that order. Set the remainder under the order added.

EXAMPLE.

£	s.	d.
5 „	6 „	8
4 „	9 „	9
9 „	9 „	5
19 „	5 „	10

Let the pupil add thus: 5 pence added to 9 are 14, and 8 are 22 pence. This sum contains 1 of the order of shillings, to be carried to that order, and 10 to be written under the order added. One shilling carried to 9 makes 10, and 9 are 19, and 6 are 25 shillings. This sum contains 1 of the order of pounds, to be carried to that order, and 5 of the order of shillings, to be written under that order. 1 pound carried to 9 makes 10, and 4 are 14, and 5 are 19 pounds, which are written under that order.

Accustom the pupils to add in this manner; also require them to separate their orders in Compound Addition by *double commas*, as in the above sum. Add the following sums:

STERLING MONEY.

	£	s.	d.	£	s.	d.
	14 „	9 „	9	13 „	10 „	2
	16 „	6 „	5	10 „	17 „	3
	18 „	12 „	11	8 „	8 „	7
Ans.	49 „	19 „	1	32 „	16 „	0

TROY WEIGHT.

	lbs.	*oz.*	*pwt.*	*oz.*	*pwt.*	*gr.*
	4 „	4 „	16	10 „	16 „	8
	8 „	8 „	19	8 „	17 „	21
	6 „	9 „	14	6 „	8 „	23
Ans.	19 „	11 „	9	26 „	3 „	4

What is the rule for compound addition? Add the sums in the manner given above.

Those pupils who have not practised the rule of simple division, may omit the following exercise and begin at Addition of Vulgar Fractions.

EXERCISES FOR OLDER PUPILS.

AVOIRDUPOISE WEIGHT.

cwt.	*qr.*	*lb.*	*lb.*	*oz.*	*dr.*
2	3	27	24	13	14
1	1	17	17	12	11
4	2	26	26	12	15
6	1	13	16	8	7

APOTHECARIES WEIGHT.

ʒ	℈	*gr.*	℥	ʒ	℈
9	1	17	10	7	2
3	2	9	6	3	0
6	1	14	7	6	1
4	0	16	9	5	2

CLOTH MEASURE.

yd.	*qr.*	*na.*	*E. E.*	*qr.*	*na.*
71	3	3	44	3	2
13	2	1	49	4	3
16	0	1	06	2	3
42	3	3	84	4	1

DRY MEASURE.

pk.	*qu.*	*pt.*	*bu.*	*pk.*	*qu.*
1	7	1	17	2	5
2	6	0	34	2	7
1	5	0	13	3	6
2	4	1	16	3	4

WINE MEASURE.

gal.	*qt.*	*pt.*	*hhd.*	*gal.*	*qt.*
39	3	1	42	61	3
17	2	1	27	39	2
24	3	0	9	14	0
19	0	0	16	24	1

LONG MEASURE.

yds.	*ft.*	*in.*	*m.*	*fur.*	*po.*
4 „	2 „	11	46 „	4 „	16
3 „	1 „	8	58 „	5 „	23
1 „	2 „	9	9 „	6 „	34
6 „	2 „	10	17 „	4 „	18

LAND, OR SQUARE MEASURE.

acres.	*roods.*	*rods.*	*sq. ft.*	*sq. in.*
478 „	3 „	31	13 „	142
816 „	2 „	17	16 „	26
49 „	1 „	27	3 „	66
63 „	3 „	34	14 „	84

SOLID MEASURE.

ton.	*ft.*	*cords.*	*ft.*
41 „	43	3 „	122
12 „	43	4 „	114
49 „	6	7 „	83
4 „	27	10 „	127

TIME.

y.	*m.*	*w.*	*h.*	*min.*	*sec.*
57 „	11 „	3	23 „	54 „	32
3 „	9 „	2	12 „	40 „	24
29 „	8 „	2	14 ,	00 „	17
46 „	10 „	2	8 „	16 „	13

CIRCULAR MOTION.

S.	°	′	°	′	″
3 „	29 „	17	29 „	59 „	50
1 „	6 „	10	00 „	40 „	10
4 „	18 „	17	4 „	10 „	49
6 „	14 „	18	11 „	6 „	10

COMPOUND SUBTRACTION.

A man has 5 yds. 3 quarters of cloth, and cuts off 2 yds. 1 qr. how much is left?

A man has 6 lbs. 3 oz. of beef, and sells 4 lbs. 2 oz. how much is left?

If 4 bushels, 3 pecks, are taken from 8 bushels, 1 peck, how many remain?

A man has 12 bushels 2 pecks of grain, and sells 7 bushels 3 pecks, how many will remain?

If 4 yards, 3 quarters, 2 nails, be taken from 6 yds. 4 qrs. 3 nails, how many will remain?

If 4 £ ,, 3*s.* ,, 4*d.* be subtracted from £6 ,, 8*s.* 5*d.* how many will remain?

If the same quantity be added to the minuend and subtrahend, is the remainder altered?

Can you add a certain quantity to the minuend in one order, and the same quantity to the subtrahend in another order? Give an example.

If you wish to subtract 1 yd. 3 quarters, from 5 yds. 2 qrs. can you subtract the 3 qrs. from the 2 qrs.?

What can you do to get the right answer?

If 4 shillings, 4 pence, be taken from 6 shillings 3 pence, how many will remain?

In which order is the subtrahend larger than the minuend? Can 4 pence be taken from 3 pence? What must you do in order to subtract?

From 10 lbs. 8 oz. subtract 9 lbs. 9 oz.

In which order is the subtrahend larger than the minuend? What must be done in this case?

From 7 feet 4 inches, subtract 5 feet 6 inches.

In which order is the subtrahend larger than the minuend? What must be done in this case?

RULE FOR COMPOUND SUBTRACTION.

Write the subtrahend under the minuend, placing units of the same order under each other. Subtract each order of the subtrahend, from the same order of the minuend, and set the remainder under. If in any order the subtrahend is larger than the minuend, add as many units to the minuend as make one of the next higher order; then add one unit to the next higher order of the subtrahend.

What is the rule for compound subtraction?

EXAMPLES.

Subtract 29£ 19*s.* 8*d.* from 36£ 15*s.* 7*d.*

Placing them according to rule they stand thus:

£.	s.	d.
36	15	7
29	19	8
6	15	11

Subtract thus: 8 shillings cannot be taken from 7; therefore add as many units of this order to 7, as are required to make one unit of the next higher order; that is, 12 (as 12 pence make one shilling). 12 added to 7 are 19. Subtract 8 from 19, and 11 remain to be set down.

As 12 pence have been added to the minuend, an equal quantity must be added to the subtrahend; therefore carry 1 shilling to the 19, which makes 20. This cannot be subtracted from 15; therefore add to the 15 as many of this order, as are required to make one unit of the next higher order; that is 20. This being added to 15 makes 35. Subtract 20 from 35, and 15 remain to be set down; as 20 shillings have been added to the minuend, 1 pound must be carried to the subtrahend of the next higher order, which makes it 30; and this subtracted from 36, leaves 6 to be written under that order.

Let the following sums be explained as above.

STERLING MONEY.

£	s.	d.		s.	d.	qrs.
44	10	2		16	8	2
36	11	8		10	7	3

TROY WEIGHT.

lb.	oz.	pwt.		oz.	pwt.	gr.
6	11	14		4	19	21
2	3	16		2	14	23

AVOIRDUPOIS WEIGHT.

c.	qr.	lb.		lb.	oz.	dr.
7	3	13		8	9	12
5	1	15		6	12	9

APOTHECARIES WEIGHT.

ʒ	℈	grs.		℥	ʒ	℈
4	1	17		10	3	1
1	2	15		7	6	1

Perform the operation by the method given above.

CLOTH MEASURE.

yd.	*qr.*	*na.*	*E. E.*	*qr.*	*na.*
35 „	1 „	2	67 „	3 „	1
19 „	1 „	3	21 „	3 „	2

DRY MEASURE.

bu.	*pk.*	*qt.*	*pk.*	*qt.*	*pt.*
65 „	1 „	7	2 „	3 „	0
14 „	3 „	4	1 „	6 „	1

WINE MEASURE.

gal.	*qt.*	*pt.*	*hhd.*	*gal.*	*qt.*
21 „	2 „	0	13 „	0 „	1
14 „	2 „	1	10 „	60 „	3

LONG MEASURE.

yd.	*ft.*	*in.*	*m.*	*fur.*	*po.*
4 „	2 „	11	41 „	6 „	22
2 „	2 „	11	10 „	6 „	23

LAND OR SQUARE MEASURE.

A.	*roods.*	*rods.*	*A.*	*r.*	*po.*
29 „	1 „	10	29 „	2 „	17
24 „	1 „	25	17 „	1 „	36

SOLID MEASURE.

tons.	*ft.*	*cords.*	*ft.*
116 „	24	72 „	114
109 „	39	41 „	120

TIME.

yrs.	*mo.*	*we.*	*h.*	*min.*	*sec.*
54 „	11 „	3	20 „	41 „	20
43 „	11 „	3	17 „	49 „	19

CIRCULAR MOTION.

S.	°	′	°	′	″
9 „	23 „	45	29 „	34 „	54
3 „	7 „	40	19 „	40 „	36

COMPOUND MULTIPLICATION.

If 4 grains, 3 penny-weights, are repeated 3 times, what is the product?

If 3 yards, 1 quarter, be repeated 3 times, what is the product?

If 5 feet, 2 inches, be repeated 4 times, what is the product?

If 2 hogsheads, 5 gallons, be repeated 5 times, what is the product?

If 4 drams, 2 ounces, be repeated 3 times, what is the product?

What is 4 times 2 days, 7 hours?

What is 5 times 3 months, 4 days?

RULE FOR COMPOUND MULTIPLICATION.

Place the multiplier below the multiplicand. Multiply each order separately, beginning with the lowest. In the product of each order, find how many units there are of the next higher order. Carry these units to the next product, and set the remainder under the order multiplied.

£.	s.	d.
1 „	9 „	6
		4
5 „	18 „	0

Proceed thus:—Four times six pence are 24 pence, which is 2 units of the next higher order, (or shillings,) to be carried to that order; and as no pence remain, a cipher is to be placed in the order of pence. Four times 9 shillings are 36 shillings, and the 2 carried make 38 shillings, which is 1 pound, to be carried to the next product, and 18 shillings to be written in the shilling order. Four times 1 pound is 4 pounds, and the 1 carried, makes 5, which is to be written in the order of pounds.

Let the pupil do the following sums, stating the process while doing it, as above.

What cost 9 yards of cloth, at 5s. 6d. per yard?

What cost 5 cwt. of raisins, at £1 3. 3d. per cwt.?

What cost 4 gallons of wine, at 8s. 7d. per gallon?

What is the weight of 6 chests of tea, each weighing 3 cwt. 2 qrs. 9 lbs.?

What is the weight of 7 hogsheads of sugar, each weighing 9 cwt. 3 qrs. 12 lbs.?

What is the rule for compound multiplication?

How much brandy in 9 casks, each containing 41 gals. 3 qts. 1 pt.?

		ANSWERS.
	yds. qr. na.	*yds. qr. na.*
1. Multiply	14 3 2 by 11	163 2 2
	hhd. g. qt. pt.	*hhd. g. qt. pt.*
2. Multiply	21 15 2 1 by 12	254 61 2 0
	le. m. fur. po.	*le. m. fur. po.*
3. Multiply	81 2 6 21 by 8	655 1 4 8
	a. r. p.	*a. r. p.*
4. Multiply	41 2 11 by 18	748 0 38
	yr. m. w. d.	*yr. m. w. d.*
5. Multiply	20 5 3 6 by 14	286 11 2 0
	S. ° ′ ″	*S.* ° ′ ″
6. Multiply	1 15 48 24 by 5	7 19 2 0

1. In 35 pieces of cloth, each measuring 27¾ yds. how many yards? *Ans.* 971 *yds.* 1 *qr.*

2. In 9 fields, each containing 14 acres, 1 rood, and 25 poles, how many acres? *Ans.* 129 *a.* 2 *roods*, 25 *rods.*

3. In 6 parcels of wood, each containing 5 cords and 96 feet, how many cords? *Ans.* 34 *cords*, 64 *feet.*

4. A gentleman is possessed of 1½ dozen of silver spoons, each weighing 2 oz. 15 pwt. 11 grs., 2 dozen of tea-spoons, each weighing 10 pwt. 14 grs., and 2 silver tankards, each 21 oz. 15 pwt. Pray what is the weight of the whole?

Ans. 8 *lb.* 10 *oz.* 2 *pwt.* 6 *grs.*

COMPOUND DIVISION.

Divide £4 ,, 8*s.* by 2. Divide £6 ,, 12*s.* by 3.

If 2 dresses contain 24 yds. 2 qrs. how much in each dress?

If 3 silver cups weigh 9 lbs. 6 oz. what is the weight of each?

In division we find how often one number is contained in another, and thus what part of one number is another. Thus if we divide 8 lbs. 16 oz. by 4, we can either say how *many times* 4 is contained in 8 and in 16, or we can say what is *one fourth* of 8 lbs. and 16 oz.

If there is any *remainder* in dividing one order, it must be changed to units of the next lower order and added to it and then divide again.

In doing the sum we place the figures thus:

	£	s.	d.
3)	4 "	18 "	9
	1 "	12 "	11

We proceed thus in explaining the process.

A third of £4 is £1 which is set under that order, and there is £1 remaining which is changed to 20 shillings and added to the 18, making 38. A third of 38 shillings is 12 shillings, which are set under that order. 2 shillings remain, which are changed to 24 pence and added to the 9 pence, making 33 pence; a third of 33 pence is 11 pence, which are set in that order.

Let the following sums be performed and explained as above.

Divide 22£ 11*s*. 6*d*. by 6.

At 2£ 8*s*. 6*d*. for 6 pair of shoes, what is that a pair?

If 9 silver cups weigh 3 lbs. 6 oz. 8 pwt. 3 grs. what is the weight of each?

If 8 dresses contain 59 yds. 3 qrs. 2 n. how much in each dress?

If the divisor exceeds 12 and is a *composite* number, divide the sum by *one* of the factors as above and the answer by the other.

EXAMPLES.

Divide £2 " 8s. " 11d. " 4 qr. by 44.

If 18 gal. " 6 qr. " 4 g. of brandy be divided equally into 28 bottles, how much does each contain?

If 24 coats contain 62 yds. 3 qrs. 4 na. how much does each contain?

If 32 teams be loaded with 40 T. 16 cwt. 3 qrs. how much is that for each team?

If the divisor exceeds 12 and is *not a composite number*, the following method is used. Let the figures be placed thus:

We first divide the pound order and 4 is the quotient figure, which is of the *pound* order because the dividend is pounds. This is put in the quotient with the £ put over it to indicate its order.

In order to find the remainder we subtract the *product of the quotient and divisor* from the 461.

```
      £    s.   d. £  s. d.
139)461 " 11 " 11(3 " 6 " 5
    417
    ---
     44
     20
    ---
    891
    834
    ---
     57
     12
    ---
    695
    695
```

The remainder is 44£. This must be changed to shillings, which is done by multiplying it by 20 and then the 11 shillings of the dividend are added.

This sum is then divided by 139 and the quotient figure is 6, which is of the *shilling* order and must be put in the quotient under that sign. Proceed as before till the orders are all thus divided.

Let the following examples be performed and explained as above.

Divide 239£ " 16*s*. " 4*d*. " 3*qr*. by 123.

If 239 yds. of cloth cost 49£ 19*s*. 11*d*. what was that per yard?

NOTE.—Change the pounds to shillings first.

If 349 cwt. 3 qrs. 12 lbs. are contained in 264 barrels, how much is in each barrel.

If 42 cwt. of tobacco cost 826£ 18*s*. 9*d*. what is that per lb.?

RULE FOR COMPOUND DIVISION.

If the divisor does not exceed 12, divide each order separately, beginning with the highest, remembering to make the quotient figure of the same order as the dividend.

Whenever there is a remainder, change it and add it to the next lower order and divide as before.

If the divisor exceeds 12, either resolve it into factors and divide first by one and then by the other, or proceed after the manner of long Division.

	T.	*cwt.*	*lb.*	*oz.*	*dr.*			
Divide	29 "	13 "	25 "	12 "	13 by 6.			
	lb.	*oz.*	*pwt.*	*grs.*				
Divide	7 "	10 "	15 "	2 by 5.				
	yds.	*qrs.*	*na.*					
Divide	76 "	3 "	2 by 4.					
	deg.	*m.*	*fur.*	*pol.*	*ft.*	*in.*	*bar.*	
Divide	97 "	55 "	7 "	35 "	4 "	2 "	1 by 7.	
	£	*s.*	*d.*	*qrs.*				
Divide	25 "	16 "	10 "	3 by 9.				

What is the rule for compound division when the divisor does not exceed 12? What is the rule, if it exceed 12?

ADDITION OF VULGAR FRACTIONS.

Sums for Mental Exercise.

If one boy has one half an orange, and another three halves, and another four halves, how many halves are there in all?

If one third of a dollar, five thirds, and six thirds, be added together, how many are there in all?

One man owns four twentieths of a building, another six twentieths, and another eight twentieths, how many twentieths do all own?

Seven thirtieths, nine thirtieths, and six thirtieths, are how many?

Eight twenty-fifths, four twenty-fifths, and seven twenty-fifths, are how many?

RULE FOR ADDING VULGAR FRACTIONS, WHEN ALL HAVE THE SAME OR A COMMON DENOMINATOR.

Add the numerators, and place their sum over the common denominator.

EXAMPLE.

Add $\frac{3}{75}$ $\frac{6}{75}$ $\frac{2}{75}$ and $\frac{4}{75}$.

The sum of the numerators is 15, which being placed over the common denominator, gives the answer $\frac{15}{75}$.

Add the following sums, using the *signs*, thus:

Add $\frac{2}{20}$ $\frac{4}{20}$ and $\frac{9}{20}$. Ans. $\frac{2}{20}+\frac{4}{20}+\frac{9}{20}=\frac{15}{20}$.

Add $\frac{3}{15}$ $\frac{8}{15}$ and $\frac{10}{15}$. Add $\frac{9}{26}$ $\frac{8}{26}$ $\frac{4}{26}$ and $\frac{2}{26}$.

Add $\frac{7}{24}$ $\frac{9}{24}$ and $\frac{8}{24}$. Add $\frac{3}{5}$ $\frac{4}{5}$ $\frac{9}{5}$.

When fractions having a *different* denominator, are added, it is necessary to perform a process which will be explained hereafter.

Those fractions which have the numerator larger than the denominator, are called *improper* fractions, thus: $\frac{10}{7}$ $\frac{7}{2}$.

When we use the expression *seven halves*, we do not mean seven halves of *one* thing, because nothing has more than two halves. But if we have seven apples, and take a half from each one, we shall have *seven halves;* and they are halves of *seven things*, and must be written as above.

What is the rule for adding vulgar fractions? What is meant by the expression *seven halves*?

SUBTRACTION OF VULGAR FRACTIONS.

If a boy has 6 ninths of an apple, and gives away 4 ninths, how much remains?

If he has 8 ninths, and gives away 5 ninths, what remains?

If he has 7 twelfths, and gives away 4 twelfths, what remains?

In doing these sums let the pupil tell first which is the minuend and which the subtrahend.

A man has 9 twentieths of a dollar and loses 5 twentieths, how much remains?

If he has 11 twentieths and loses 7 twentieths, what remains?

If he has 8 sixteenths, and loses 5 sixteenths, what remains?

Subtract $\frac{3}{12}$ from $\frac{9}{12}$. Subtract $\frac{8}{20}$ from $\frac{12}{20}$.

RULE FOR SUBTRACTING VULGAR FRACTIONS.

Subtract the numerator of the subtrahend, from the numerator of the minuend, and place the remainder over the common denominator.

Let the pupil, in doing the sums, use the *signs* in this way.

Subtract $\frac{6}{8}$ of a dollar from $\frac{9}{8}$.

Ans. $\frac{9}{8} - \frac{6}{8} = \frac{3}{8}$.

Subtract $\frac{6}{20}$	from $\frac{9}{20}$.	Subtract $\frac{35}{64}$	from $\frac{49}{64}$.
" $\frac{64}{900}$	" $\frac{201}{900}$	" $\frac{210}{480}$	" $\frac{320}{480}$.
" $\frac{16}{80}$	" $\frac{19}{80}$	" $\frac{16}{30}$	" $\frac{20}{30}$.
" $\frac{96}{5000}$	" $\frac{120}{5000}$	" $\frac{4}{20}$	" $\frac{12}{20}$.

A man owns $\frac{8}{5}$ of a pasture, and sells $\frac{3}{5}$, how much remains his own?

A boy has $\frac{14}{20}$ of a guinea, and gives away $\frac{6}{20}$, how much has he left?

$\frac{9}{40}$ from $\frac{25}{40}$, are how many? $\frac{10}{60}$ from $\frac{21}{60}$ are how many?

$\frac{18}{50}$ from $\frac{36}{50}$ are how many? $\frac{7}{25}$ from $\frac{15}{25}$? $\frac{8}{18}$ from $\frac{18}{18}$?

What is the rule for subtracting Vulgar Fractions?

MULTIPLICATION OF VULGAR FRACTIONS.

MULTIPLICATION WHEN ONLY THE MULTIPLICAND IS A FRACTION.

A man gave one child three quarters of a dollar, and another four times as much, how much did he give the last?

A man has 12 barrels of wine, and takes a half pint from each 3 times, how many half pints does he take?

If a man has an ounce of silver, and takes 2 sixteenths from it 6 times, how many sixteenths does he take?

How much is 4 times two sixths?

How much is 5 times two sixths? 6 times? 7 times?

From the above examples it appears, that we can multiply a fraction by a whole number, by *multiplying its numerator.*

Let the pupil perform the following sums, first mentally, and then on the slate.

1. What is 9 times $\frac{2}{20}$?
2. What is 3 times $\frac{4}{18}$?
3. What is 6 times $\frac{3}{25}$?
4. What is 7 times $\frac{6}{40}$?
5. What is 8 times $\frac{5}{60}$?
6. What is 7 times $\frac{11}{100}$?
7. What is 3 times $\frac{9}{30}$?
8. What is 5 times $\frac{8}{65}$?
9. What is 8 times $\frac{6}{81}$?
10. What is 4 times $\frac{8}{74}$?
11. What is 6 times $\frac{6}{40}$?
12. What is 5 times $\frac{3}{20}$?
13. What is 4 times $\frac{3}{16}$?
14. What is 8 times $\frac{8}{96}$?
15. What is 5 times $\frac{6}{85}$?
16. What is 6 times $\frac{3}{39}$?
17. What is 4 times $\frac{8}{56}$?
18. What is 3 times $\frac{9}{33}$?
19. What is 9 times $\frac{4}{58}$?
20. What is 6 times $\frac{5}{80}$?

In performing these sums on the slate, let the pupil use the *signs*, thus:

Two twentieths multiplied by nine, equals eighteen twentieths; and is expressed by signs as follows:

$$\frac{2}{20} \times 9 = \frac{18}{20}.$$

There is another method, by which the *value* of a fraction is multiplied, by increasing the *size* of the parts expressed by the denominator.

How can a fraction be multiplied by a whole number? What is the second method of increasing the value of a fraction?

For example, when we wish to multiply $\frac{4}{12}$ by 2, the most common way is to multiply the numerator by 2, thus:

$$\tfrac{4}{12} \times 2 = \tfrac{8}{12}.$$

But the same *effect* is produced, if we *divide the denominator* by 2, thus:

$$\tfrac{4}{12} \times 2 = \tfrac{4}{6}.$$

It will easily be seen, that $\frac{8}{12}$ and $\frac{4}{6}$ are the *same quantity.* The only difference is, that in one case the unit is divided into 12 parts and 8 are expressed, and in the other case, the unit is divided into 6 parts, and 4 are expressed. In one case, we make *twice as many* pieces, and in the other we make them *twice as large.*

When we multiply the numerator, the *number of parts* is multiplied, and when we divide the denominator the *size of the parts* is multiplied.

If we multiply $\frac{4}{12}$ by 3, in what two ways can it be done?

If we multiply the numerator, what is it that is multiplied?

If we divide the denominator, what is it that is multiplied?

Multiply $\frac{6}{9}$ by 3 in both ways, and tell what each method multiplies.

RULE FOR MULTIPLYING WHEN ONLY THE MULTIPLICAND IS A FRACTION.

Multiply the numerator, or divide the denominator by the multiplier.

Let the following sums be performed, and explained as above.

Multiply	$\frac{3}{16}$	by	4	Multiply	$\frac{3}{8}$	by	2
"	$\frac{4}{18}$	"	9	"	$\frac{3}{42}$	"	7
"	$\frac{4}{54}$	"	6	"	$\frac{6}{45}$	"	7
"	$\frac{5}{27}$	"	9	"	$\frac{4}{64}$	"	8
"	$\frac{5}{30}$	"	10	"	$\frac{5}{15}$	"	3
"	$\frac{6}{35}$	"	7	"	$\frac{6}{80}$	"	10
"	$\frac{9}{40}$	"	5	"	$\frac{7}{50}$	"	5
"	$\frac{6}{40}$	"	8	"	$\frac{3}{56}$	"	8

What is the difference between the two methods? What is the rule for multiplying when the *multiplicand* only is a fraction?

Multiply	$\frac{4}{12}$	by	6	Multiply	$\frac{3}{48}$	by	6
"	$\frac{6}{81}$	"	9	"	$\frac{4}{55}$	"	11

MULTIPLICATION WHERE ONLY THE MULTIPLIER IS A FRACTION.

1. If you have twelve cents, and give away a *sixth* of them to each of *four* children, how many cents do you give away?

Ans. A *sixth* of twelve cents is *two* cents. *Two* cents given to each of *four* children would be eight cents given away.

2. If a man has fifteen cents, and gives a fifth of them to each of *three* children, how many does he give away?

Ans. One fifth of fifteen is three. Three times three is nine. He gives away *nine cents*.

From the above examples it appears that when we *multiply by a fraction*, we take a *part* of the multiplicand, and repeat it a certain number of times. In the last case the man had fifteen cents, which is the multiplicand. We take a *fifth* of it and repeat it *three* times.

3. If a man had eighteen cents, and gave a *ninth* of them to *six* different boys, how many cents did he give away?

In the above question, what is the multiplicand? What part are you to take from it, and how often are you to repeat it?

4. If a man has *twelve* dollars, and gave a *fourth* of them to *three* different workmen, how many did he give away? What is the multiplicand? What part are you to take from it, and how often are you to repeat it?

5. How do you multiply *twelve* by *three fourths*?

Ans. We take a fourth of twelve and repeat it three times. *One fourth* of twelve is *three*. *Three fourths* are three times as much. Three times three is nine.

6. How do you multiply *eight* by *three fourths*?

7. How do you multiply *eighteen* by *three ninths*?

8. If you multiply *twelve* by *three*, do you make it larger or smaller? If you multiply it by three fourths, do you make it larger or smaller?

Why is the multiplicand made smaller when you multiply by *three fourths*?

Ans. Because we do not repeat the *whole* number, but only a *fourth part* of it; and this is repeated only three times, which does not make it as large a number as the multiplicand.

9. If you multiply eight by three, do you make it larger or

smaller? If you multiply it by *three fourths*, do you make it larger or smaller? Why?

10. Multiply *fifteen* by *two thirds*.
11. Multiply *twenty-four* by *five sixths*.
12. Multiply *thirty-two* by *three eighths*.
13. Multiply *fourteen* by *three sevenths*.
14. Multiply *sixteen* by *two eighths*.
15. Multiply *twenty-four* by *five sixths*.

Multiplication has been defined, as repeating a number, as often as there are units in another number.

In multiplying by a fraction, we take such *a part* of a number, as is expressed by the *denominator*, and repeat it as often as there are units in the *numerator*.

Thus in multiplying 12 by $\frac{4}{6}$ we take a sixth part of 12, and repeat it 4 times, and the answer is 8.

NOTE.—The propriety of calling the number in the numerator *units*, is explained on page 34, where the distinction is shown between units that are whole numbers, and units that are fractions. It is shown also on page 49, where it appears that the numerator may be considered as whole numbers, divided by the denominator.

In multiplying let the pupil use the signs thus:

Multiply 12 by $\frac{3}{6}$. $12 \div 6 = 2$.
$2 \times 3 = 6$. Answer.

In doing the above sum what part of 12 is taken? How often is it repeated?

Is the product larger or smaller than the multiplicand?

Multiply 12 by $\frac{9}{6}$.

Is $\frac{9}{6}$ a proper or improper fraction?

Is there a whole unit in $\frac{9}{6}$?

Is the product larger or smaller than the multiplicand, when 12 is multiplied by $\frac{9}{6}$?

Why is it larger when multiplied by $\frac{9}{6}$ and smaller when multiplied by $\frac{3}{4}$?

Let the following sums be stated thus; $16 \times \frac{2}{4}$. *One* fourth of 16 is 4, and *two* fourths, is twice as much, or 8.

How do we multiply by a fraction?

Multiply	16	by $\frac{1}{4}$	by	$\frac{3}{4}$	by	$\frac{8}{4}$
"	18	" $\frac{1}{6}$	"	$\frac{2}{6}$	"	$\frac{9}{6}$
"	24	" $\frac{1}{8}$	"	$\frac{7}{8}$	"	$\frac{12}{8}$
"	36	" $\frac{1}{9}$	"	$\frac{7}{9}$	"	$\frac{8}{9}$
"	42	" $\frac{1}{7}$	"	$\frac{4}{7}$	"	$\frac{12}{7}$
"	63	" $\frac{1}{9}$	"	$\frac{5}{9}$	"	$\frac{8}{9}$

EXAMPLES FOR MENTAL EXERCISE.

If you have 14 apples, and give one seventh of them to each of four boys, how many do you give away?

Ans. A seventh to 1 boy, would be 2, and four sevenths, would be four times as much, or 8.

What is $\frac{4}{7}$ of 14?

If you have 48 cents, and give a twelfth of them, to each of two boys, how many do you give away?

What is $\frac{2}{12}$ of 48?

A man has 35 sheep, and sells four fifths of them, how many does he sell?

A boy has 40 marbles, and loses $\frac{5}{8}$ of them, how many does he lose?

What is 40 multiplied by $\frac{5}{8}$?

What is 36 multiplied by $\frac{6}{12}$?

What is $\frac{3}{7}$ of 21? $\frac{4}{6}$ of 24? $\frac{8}{9}$ of 81? $\frac{4}{7}$ of 49? $\frac{5}{8}$ of 64? $\frac{2}{4}$ of 16? $\frac{6}{5}$ of 40? $\frac{8}{9}$ of 45? $\frac{3}{10}$ of 60? $\frac{5}{12}$ of 96? $\frac{7}{4}$ of 24? $\frac{8}{5}$ of 30?

What is $\frac{2}{9}$ of 18? $\frac{4}{20}$ of 100? $\frac{6}{5}$ of 40? $\frac{3}{7}$ of 28? $\frac{5}{9}$ of 27? $\frac{8}{11}$ of 33? $\frac{6}{8}$ of 48? $\frac{5}{9}$ of 81? $\frac{8}{12}$ of 144? $\frac{10}{11}$ of 99?

What is $\frac{2}{6}$ of 54? $\frac{3}{7}$ of 49? $\frac{4}{8}$ of 32? $\frac{5}{9}$ of 81? $\frac{6}{10}$ of 70? $\frac{7}{11}$ of 88? $\frac{3}{12}$ of 96? $\frac{5}{4}$ of 16? $\frac{3}{6}$ of 12? $\frac{8}{9}$ of 18? $\frac{9}{12}$ of 24? $\frac{6}{5}$ of 15? $\frac{3}{12}$ of 36?

EXAMPLES FOR THE SLATE, FOR OLDER PUPILS, WHO UNDERSTAND THE RULE OF DIVISION.

$1122 \times \frac{8}{14}$	$1912 \times \frac{19}{24}$
$144 \times \frac{6}{12}$	$1357 \times \frac{13}{24}$

$2608 \times \frac{12}{15}$	$545 \times \frac{16}{34}$
$720 \times \frac{9}{20}$	$722 \times \frac{5}{19}$
$1335 \times \frac{12}{15}$	$304 \times \frac{18}{10}$
$578 \times \frac{8}{17}$	$420 \times \frac{9}{13}$

If a number is to be both multiplied and divided by two figures, it makes no difference which is done *first*, provided the same figures are used as multiplier and divisor.

For example, let a number be *multiplied* by 2, and *divided* by 9.

We can divide first by 9, and then multiply the quotient by 2; or we can multiply first by 2, and then divide the product by 9, and the answer is the same.

Thus 18 multiplied by 2, is 36; and this divided by 9 is 4.

Again 18 divided by 9, is 2; and this multiplied by 2 is 4.

If then we multiply 12 by $\frac{3}{4}$ we divide by 4, to find *one* fourth of 12, and multiply by 3, to obtain *three* fourths, and the answer is 9. But if we should multiply 12 by 3, and then divide the product by 4, the answer would be the same. Thus $12 \times 3 = 36$ and $36 \div 4 = 9$. Thus 9 is the same answer as is obtained by dividing 12 by the denominator, and multiplying the answer, by the numerator. What are the two ways in which 18 can be multiplied by 4-6? What will be the answer, if it is divided by 6 first, and the quotient multiplied by 4? What will be the answer, if it is multiplied by 4 first, and then the product divided by 6?

RULE FOR MULTIPLYING WHEN ONLY THE MULTIPLIER IS A FRACTION.

Divide by the denominator, to obtain one part, and multiply by the numerator, to obtain the required number of parts.

But in case this division should leave a remainder;

Multiply by the numerator first, and then divide the product by the denominator.

EXERCISES FOR THE SLATE, FOR OLDER PUPILS.

In all these cases it is best to *multiply by the numerator first*, and then divide by the denominator. If any remains after division, place the divisor under it, for a fraction.

What is the rule for multiplying when the *multiplier only* is a fraction?

Multiply,	1369 by 3-6	Multiply	4681 by 9-4
"	5436 " 5-8	"	3642 " 5-8
"	3264 " 2-5	"	5963 " 6-2
"	43256 " 3-6	"	46938 " 5-9
"	86432 " 8-4	"	63921 " 4-12
"	3549 " 8-9	"	26438 " 6-5
"	54683 " 5-8	"	39621 " 5-8

EXAMPLES FOR MENTAL EXERCISE.

1. If 15 is *five* eighths of some number, what part of 15 is *one* eighth of that number?

2. If 12 is *four* sixths of some number, what part of 12 is *one* sixth of that number?

3. If 18 is *six* ninths of some number, what part of 18 is one *ninth* of that number?

4. If 15 is $\frac{5}{8}$ of a number, what is $\frac{1}{8}$ of that number?

5. If 15 is $\frac{5}{8}$ of some number, what is that number?

Let such exercises be stated thus.

6. If 15 is *five* eighths, a *fifth* of 15 is *one* eighth. A fifth of 15 is 3. If 3 is *one* eighth, then the whole is 8 times as much, or 24.

7. 24 is $\frac{3}{8}$ of what number?

8. 36 is $\frac{9}{8}$ of what number?

9. 42 is $\frac{7}{9}$ of what number?

10. If a man can do $\frac{6}{7}$ of a piece of work in 12 days, how long would it take him to do $\frac{1}{7}$ of it?

Ans. It would take him only *one sixth* of the time to do *one seventh* that it does to do $\frac{6}{7}$. $\frac{1}{6}$ of 12 is 2. It would take him 2 days.

Let the remaining sums be stated as above.

11. If a man bought $\frac{3}{8}$ of a barrel of wine for 18 dollars, how much will $\frac{1}{6}$ cost?

12. How much will the whole cost?

13. Bought $\frac{6}{8}$ of a chaldron of coal for 24 shillings, how much will $\frac{1}{8}$ cost? How much will the whole cost?

14. If 15 is $\frac{5}{8}$ of some number, what is one eighth of that number.

15. What is the whole of that number? If 23 is $\frac{7}{9}$ of one number, what is that number?

16. If a man bought $\frac{6}{7}$ of a cask of brandy for 42 dollars, what is $\frac{1}{7}$ worth? what is the whole worth?

17. If $\frac{4}{5}$ of a month's board cost 12 dollars what is it a month?

18. If $\frac{8}{9}$ of a cord of wood cost 16 shillings, what would $\frac{1}{9}$ cost, and what would the whole cost?

19. 28 is $\frac{7}{8}$ of what number?

20. 48 is $\frac{3}{4}$ of what number?

21. 56 is $\frac{8}{9}$ of what number?

22. 32 is $\frac{8}{10}$ of what number?

23. How many times is 4 contained in 5?

Ans. Once and one over.

24. What is $\frac{1}{4}$ of 1? What is $\frac{1}{6}$ of 1?

25. What is $\frac{1}{3}$ of 1? What is $\frac{1}{3}$ of 2? What is $\frac{2}{3}$ of 2?

26. What is $\frac{1}{3}$ of 12? What is $\frac{1}{5}$ of 1? What is $\frac{1}{5}$ of 2? What is $\frac{1}{5}$ of 4? What is $\frac{3}{5}$ of 4?

27. What is $\frac{2}{5}$ of 4? What is $\frac{4}{5}$ of 2? What is $\frac{3}{5}$ of 1?

28. How many units in $\frac{8}{6}$ of 2?

29. How many units in $\frac{9}{4}$ of 3?

30. How many units in $\frac{4}{3}$ of 5?

31. How many units in $\frac{3}{7}$ of 11?

32. How many units in $\frac{5}{8}$ of 6?

33. How many units in $\frac{8}{9}$ of 18?

34. How many units in $\frac{4}{3}$ of 16?

35. How many units in $\frac{12}{8}$ of 21?

It will be seen that in fractions, as in whole numbers, it makes no difference in the product, which factor is used as *multiplier*.

For $12 \times \frac{3}{4} = 9$

And $\frac{3}{4} \times 12 = \frac{36}{4} = 9.$

Here when the *whole number* is used as multiplier, the answer is an improper fraction, which, if changed to whole numbers, is 9.

Multiply 18 by $\frac{3}{8}$ and $\frac{3}{8}$ by 18, and tell in what respects the answers differ.

Multiply $\frac{3}{7}$ by 14, and 14 by $\frac{3}{7}$.
Is there any difference in the *value* of the answers?
In what respect do they differ.

MULTIPLICATION WHERE BOTH FACTORS ARE FRACTIONS.

1. If we had $\frac{1}{2}$ an orange and should give away half of this $\frac{1}{2}$ what part of an orange should we give away?

How much is $\frac{1}{2}$ of $\frac{1}{2}$.

2. If we have $\frac{1}{2}$ of an orange, and should give away $\frac{1}{4}$ of it, what part of a whole orange should we give away?

Ans. If the two halves of any thing be divided into 4 pieces each, the *whole* is divided into 8 pieces. Taking $\frac{1}{4}$ of *one* of these halves then, is taking $\frac{1}{8}$ of the whole.

$\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$.

3. If we have $\frac{1}{4}$ of an orange, and give away *half* of it, what part of the whole orange do we give away?

Ans. If an orange is divided into 4 pieces, and each of these pieces are *halved*, the orange is divided into 8 pieces, and each piece is $\frac{1}{8}$ of the whole.

$\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$.

4. If you receive $\frac{1}{3}$ of an orange, and you give $\frac{1}{4}$ of it away, what part of the whole orange do you give away?

Ans. The orange is divided is 3 parts; if each of these parts is divided into 4 parts, the whole orange would be divided into 12 parts, and each part is $\frac{1}{12}$ of the whole.

$\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$.

5 If you have an apple and it is cut into 5 equal parts, what part of the apple is each piece? If each *piece* is cut into 3 equal parts, what part of the *whole apple* is each piece?

Ans. If an apple is cut into 5 equal parts, each part is *one fifth* of the whole, and if each of these pieces is divided into 3 parts, each part is $\frac{1}{15}$ of the whole.

6. If you have an orange, and it is divided into 3 equal parts, each part is *one third*, if each $\frac{1}{3}$ is divided into 6 equal pieces, what part of the $\frac{1}{3}$ is each piece?

7. What part of the *whole orange* is each piece?

8. If a loaf of bread is cut into 4 equal parts, each part is $\frac{1}{4}$. If each $\frac{1}{4}$ is divided into 5 equal pieces, each piece is $\frac{1}{5}$ of the $\frac{1}{4}$, and $\frac{1}{20}$ of the whole loaf, $\frac{1}{5}$ of $\frac{1}{4}$ then is $\frac{1}{20}$.

9. If a sheet of paper is cut into 5 pieces, each piece is $\frac{1}{5}$. If each $\frac{1}{5}$ is cut into 3 equal pieces, each piece is $\frac{1}{3}$ of the $\frac{1}{5}$, and $\frac{1}{15}$ of the *whole*. $\frac{1}{3}$ of $\frac{1}{5}$ then is $\frac{1}{15}$.

10. If a yard of cloth is cut into 8 equal pieces, and each piece is then cut into 3 equal parts, what part of the *whole* is each piece?

11. If a bushel of apples is divided into *fourths* of a bushel, and each *fourth* is divided into 6 equal portions, what part of the whole is each portion?

12. If you divide a pine apple into 3 equal parts, and each of those parts into 6 equal pieces, what part of the whole is each piece?

13. If you have $\frac{1}{5}$ of a dollar, and wish to give $\frac{1}{7}$ of it to each of 7 children, what part of the whole dollar do you give to each?

14. If you have $\frac{1}{8}$ of a lb. of raisins, and wish to divide it equally between 3 children, what part of a lb. do you give to each?

15. If you have $\frac{1}{6}$ of a yard of muslin, and divide it into 8 equal pieces, what part of $\frac{1}{6}$ is each piece, and what part of the *whole* yard is each piece?

16. What part of a unit is $\frac{1}{8}$ of $\frac{1}{6}$?

Ans. If a unit is divided into 6 parts, and each of these parts into 8, the unit would be divided into 48 parts, and each part is $\frac{1}{48}$ of the whole.

Let the following sums be stated in the same manner.

17. What part of a unit is $\frac{1}{5}$ of $\frac{1}{8}$?
18. What part of a unit is $\frac{1}{9}$ of $\frac{1}{4}$?
19. What part of a unit is $\frac{1}{4}$ of $\frac{1}{5}$?
20. What part of a unit is $\frac{1}{7}$ of $\frac{1}{3}$?
21. What part of a unit is $\frac{1}{6}$ of $\frac{1}{7}$?
22. What part of a unit is $\frac{1}{9}$ of $\frac{1}{8}$?
23. What part of a unit is $\frac{1}{6}$ of $\frac{1}{11}$?
24. What part of a unit is $\frac{1}{5}$ of $\frac{1}{9}$?
25. What part of a unit is $\frac{1}{3}$ of $\frac{1}{10}$?
26. What part of a unit is $\frac{1}{4}$ of $\frac{1}{8}$?
27. What part of a unit is $\frac{1}{6}$ of $\frac{1}{12}$?

28. What is $\frac{1}{3}$ of $\frac{1}{2}$?
29. What is $\frac{1}{5}$ of $\frac{1}{10}$?
30. What is $\frac{1}{8}$ of $\frac{1}{12}$?
31. What is $\frac{1}{9}$ of $\frac{1}{7}$?
32. What is $\frac{1}{8}$ of $\frac{1}{6}$?
33. What is $\frac{1}{5}$ of $\frac{1}{2}$? What is $\frac{1}{4}$ of $\frac{1}{8}$?
34. What is $\frac{1}{2}$ of $\frac{1}{6}$? What is $\frac{1}{6}$ of $\frac{1}{8}$?
35. What is $\frac{1}{7}$ of $\frac{1}{8}$? What is $\frac{1}{6}$ of $\frac{1}{7}$?
36. What is $\frac{1}{9}$ of $\frac{1}{6}$? What is $\frac{1}{6}$ of $\frac{1}{9}$?
37. What is $\frac{1}{3}$ of $\frac{1}{10}$? What is $\frac{1}{10}$ of $\frac{1}{3}$?
38. What is $\frac{1}{4}$ of $\frac{1}{9}$? What is $\frac{1}{9}$ of $\frac{1}{4}$?
39. What is $\frac{1}{7}$ of $\frac{1}{12}$? What is $\frac{1}{12}$ of $\frac{1}{7}$?
40. What is $\frac{1}{9}$ of $\frac{1}{9}$? What is $\frac{1}{7}$ of $\frac{1}{7}$?
41. What is $\frac{1}{5}$ of $\frac{1}{11}$? What is $\frac{1}{3}$ of $\frac{8}{11}$?
42. What is $\frac{1}{11}$ of $\frac{1}{9}$? What is $\frac{1}{4}$ of $\frac{1}{12}$?
43. What is $\frac{1}{3}$ of $\frac{1}{12}$? What is $\frac{1}{7}$ of $\frac{1}{6}$?

After finding $\frac{1}{4}$ of one third we know that $\frac{1}{4}$ of two thirds is twice as much.

1. What is $\frac{1}{4}$ of $\frac{1}{3}$? What is $\frac{1}{4}$ of $\frac{2}{3}$?
2. What is $\frac{1}{3}$ of $\frac{1}{5}$? What is $\frac{1}{3}$ of $\frac{2}{5}$?
3. What is $\frac{1}{5}$ of $\frac{1}{6}$? What is $\frac{1}{5}$ of $\frac{2}{6}$?
4. What is $\frac{1}{7}$ of $\frac{1}{9}$? What is $\frac{1}{7}$ of $\frac{2}{9}$?
5. What is $\frac{1}{6}$ of $\frac{1}{3}$? What is $\frac{1}{6}$ of $\frac{2}{3}$?
6. What is $\frac{1}{3}$ of $\frac{1}{6}$? What is $\frac{1}{3}$ of $\frac{2}{6}$?
7. What is $\frac{1}{3}$ of $\frac{3}{6}$? What is $\frac{1}{3}$ of $\frac{4}{6}$?
8. What is $\frac{1}{3}$ of $\frac{5}{6}$?
9. What is $\frac{1}{7}$ of $\frac{1}{9}$? What is $\frac{1}{7}$ of $\frac{2}{9}$? What is $\frac{1}{7}$ of $\frac{3}{9}$? What is $\frac{1}{7}$ of $\frac{4}{9}$? What is $\frac{1}{7}$ of $\frac{5}{9}$?
10. What is $\frac{1}{8}$ of $\frac{1}{5}$? What is $\frac{1}{8}$ of $\frac{3}{5}$?
11. What is $\frac{1}{7}$ of $\frac{1}{9}$? What is $\frac{1}{7}$ of $\frac{5}{9}$?
12. What is $\frac{1}{3}$ of $\frac{1}{12}$? What is $\frac{1}{3}$ of $\frac{7}{12}$?
13. What is $\frac{1}{9}$ of $\frac{2}{10}$? What is $\frac{1}{9}$ of $\frac{9}{10}$?
14. What is $\frac{1}{8}$ of $\frac{4}{6}$? What is $\frac{1}{9}$ of $\frac{3}{7}$?
15. What is $\frac{1}{7}$ of $\frac{4}{8}$? What is $\frac{1}{12}$ of $\frac{2}{3}$?

16. What is $\frac{1}{10}$ of $\frac{3}{7}$? What is $\frac{1}{8}$ of $\frac{6}{9}$?
17. What is $\frac{1}{3}$ of $\frac{2}{12}$? What is $\frac{1}{11}$ of $\frac{2}{10}$?
18. What is $\frac{1}{9}$ of $\frac{11}{12}$? What is $\frac{1}{2}$ of $\frac{10}{11}$?
19. What is $\frac{1}{6}$ of $\frac{4}{10}$? What is $\frac{2}{5}$ of $\frac{5}{6}$?
20. What is $\frac{1}{9}$ of $\frac{3}{9}$? What is $\frac{1}{8}$ of $\frac{5}{12}$
21. What is $\frac{1}{8}$ of $\frac{4}{6}$? What is $\frac{1}{7}$ of $\frac{7}{8}$?

After finding *one* part of a fraction, we find the other parts by multiplication.

Thus after finding what *one* fourth of a fraction is, we can find *three* fourths by multiplying by 3.

Thus $\frac{1}{4}$ of $\frac{2}{6}$ is $\frac{2}{24}$, therefore $\frac{3}{4}$ of $\frac{2}{6}$ is 3 times as much, or $\frac{6}{24}$.

1. What is $\frac{1}{4}$ of $\frac{2}{6}$? What is $\frac{2}{4}$ of $\frac{2}{6}$? What is $\frac{3}{4}$ of $\frac{2}{6}$?
2. What is $\frac{1}{5}$ of $\frac{3}{8}$? What is $\frac{2}{5}$ of $\frac{3}{8}$? What is $\frac{3}{5}$ of $\frac{3}{8}$?
3. What is $\frac{1}{9}$ of $\frac{4}{6}$? What is $\frac{2}{9}$ of $\frac{4}{6}$? What is $\frac{3}{9}$ of $\frac{4}{6}$?

Let the pupil reason thus: What is $\frac{4}{6}$ of $\frac{2}{3}$? *One* sixth of *one* third is $\frac{1}{18}$. *One* sixth of *two* thirds is $\frac{2}{18}$. *Four* sixths of two thirds is 4 times as much, or $\frac{8}{18}$.

4. What is $\frac{3}{4}$ of $\frac{8}{6}$? What is $\frac{1}{2}$ of $\frac{2}{3}$? What is $\frac{2}{3}$ of $\frac{3}{4}$?
What is $\frac{4}{3}$ of $\frac{4}{5}$? What is $\frac{3}{4}$ of $\frac{5}{6}$? What is $\frac{4}{5}$ of $\frac{6}{7}$? What is $\frac{5}{6}$ of $\frac{7}{8}$? What is $\frac{6}{7}$ of $\frac{8}{9}$?
6. What is $\frac{7}{8}$ of $\frac{9}{10}$? What is $\frac{9}{10}$ of $\frac{11}{12}$?
7. What is $\frac{11}{12}$ of $\frac{1}{2}$? What is $\frac{10}{12}$ of $\frac{2}{3}$?
8. What is $\frac{9}{10}$ of $\frac{3}{4}$? What is $\frac{8}{9}$ of $\frac{5}{6}$? What is $\frac{7}{8}$ of $\frac{6}{7}$? What is $\frac{8}{9}$ of $\frac{7}{8}$? What is $\frac{6}{7}$ of $\frac{4}{5}$?

In multiplying one fraction by another, we are to take a certain *part* of one fraction, as often as there are units in the numerator of the other fraction.

Thus, if we are to multiply $\frac{3}{4}$ by $\frac{4}{6}$ we are to take a sixth of $\frac{3}{4}$ *four times.*

To explain the *rule* for multiplying, when *both* factors are fractions, take an example. What is $\frac{3}{5}$ of $\frac{4}{6}$?

One fifth of $\frac{4}{6}$ is $\frac{4}{30}$, and this is made by multiplying the *denominator* 6, by the *denominator* 5.

In multiplying 3-5 by 4-6, what effect is produced by multiplying the numerators together?

Three fifths of $\frac{4}{6}$ is three times as much or $\frac{12}{30}$, and this is made by multiplying the *numerator* 4, by the *numerator* 3.

Therefore multiplying the *denominators* together obtained *one* fifth of $\frac{4}{6}$, and multiplying the *numerators* together, obtained *three* fifths.

RULE FOR MULTIPLYING WHEN BOTH FACTORS ARE FRACTIONS.

Multiply the denominators together to obtain one part, and the numerators together to obtain the required number of parts.

In performing these sums upon the slate, let the pupil use the signs thus:

Multiply $\frac{5}{8}$ by $\frac{3}{12}$. $\frac{5}{8} \times \frac{3}{12} = \frac{15}{96}$.

EXERCISES FOR THE SLATE, FOR OLDER PUPILS.

What is $\frac{5}{12}$ of $\frac{3}{12}$? What is $\frac{3}{9}$ of $\frac{4}{8}$? What is $\frac{5}{6}$ of $\frac{8}{9}$? What is $\frac{3}{8}$ of $\frac{4}{6}$? What is $\frac{5}{8}$ of $\frac{18}{21}$? What is $\frac{9}{12}$ of $\frac{84}{40}$? What is $\frac{18}{19}$ of $\frac{54}{60}$? What is $\frac{36}{79}$ of $\frac{121}{436}$? What is $\frac{346}{947}$ of $\frac{2460}{8964}$?

Multiply $\frac{24}{60}$ by $\frac{15}{21}$. Multiply $\frac{308}{640}$ by $\frac{64}{360}$.
Multiply $\frac{171}{200}$ by $\frac{374}{565}$. Multiply $\frac{143}{256}$ by $\frac{176}{532}$.

DIVISION OF VULGAR FRACTIONS.

DIVISION WHERE ONLY THE DIVISOR IS A FRACTION.

If we have 3 apples, how many $\frac{1}{2}$ in the whole? Ans. In *one* apple there are *two* halves, and in *three* apples there are three times as many, or *six* halves.

If we have 6 dollars, how many $\frac{1}{3}$ in the whole? Ans. In *one* dollar there are *three* thirds, and in *six* dollars there are six times as many, or *eighteen* thirds?

If we have 9 apples how many $\frac{1}{3}$?
In 8 apples how many $\frac{1}{6}$?
In 12 apples how many $\frac{1}{7}$?
In 7 apples how many $\frac{1}{12}$?

What effect is produced by multiplying the denominators together? What is the rule for multiplying when both factors are fractions?

It thus appears that when we divide by a fraction (unless it be an *improper* fraction) the quotient is *larger* than the dividend.

Thus 12 divided by $\frac{1}{4}$ is 48, for there are 48 *one fourths* in 12 *units.*

Again 9 divided by $\frac{1}{3}$ is 27, for there are 27 *one thirds* in 9 *units.*

How many $\frac{1}{5}$ in 8? How many $\frac{1}{6}$ in 12?

Divide 7 by $\frac{1}{7}$ Divide 6 by $\frac{1}{6}$ Divide 12 by $\frac{1}{3}$ Divide 10 by $\frac{1}{3}$ Divide 8 by $\frac{1}{2}$ Divide 11 by $\frac{1}{5}$ Divide 12 by $\frac{1}{12}$ Divide 9 by $\frac{1}{8}$.

How many $\frac{1}{5}$ in 8? How many $\frac{1}{9}$ in 7?

If you divide 8 by $\frac{1}{3}$ the answer is 24, for there are 24 one thirds in 8. But if we are to divide 8 by $\frac{2}{3}$ there will be but *half* as many. For there is but half as many *two* thirds as there are *one* thirds in a number. Therefore if 8 divided by $\frac{1}{3}$ is 24, when divided by $\frac{2}{3}$ it is half as much, or 12.

How many $\frac{2}{6}$ in 3?

Ans. In 3 there are 18 *one* sixth and half as many *two* sixths, or 9.

How many $\frac{3}{4}$ in 12? How many $\frac{3}{8}$ in 2?
How many $\frac{4}{7}$ in 4? How many $\frac{2}{5}$ in 6?
How many $\frac{3}{8}$ in 3?

Divide 4 by $\frac{2}{6}$ Divide 5 by $\frac{3}{8}$ Divide 3 by $\frac{3}{6}$ Divide 8 by $\frac{2}{3}$ Divide 2 by $\frac{2}{5}$ Divide 7 by $\frac{2}{6}$ Divide 5 by $\frac{2}{10}$ Divide 12 by $\frac{3}{4}$.

If you have 12 yards of long lawn and wish to cut a number of handkerchiefs of $\frac{3}{4}$ of a yard each, how many can you make from the whole piece?

If you have 4 oranges and wish to give $\frac{2}{3}$ of an orange to your mates, to how many could you give them?

If you have 4 pounds of rice to distribute to the poor, and are to give $\frac{2}{6}$ of a pound to each person, to how many persons can you give?

If a reservoir is filled by a spout in $\frac{3}{7}$ of an hour, how many times would the cistern be filled in 9 hours?

If a pound of raisins can be bought for $\frac{2}{5}$ of a dollar, how many pounds can you buy for 4 dollars?

If $\frac{3}{5}$ of a barrel of flour will last a family one week, how long will 6 barrels last?

If a cow eats $\frac{2}{7}$ of a ton of hay a month, how long would 4 tons last her?

If $\frac{5}{8}$ of a barrel of flour last a family one week, how long will 10 barrels last?

It is seen by the preceding examples, that when a number is to be divided by a fraction, it is *multiplied by its denominator*, and *divided by its numerator*.

Thus if we are to divide 2 by $\frac{3}{4}$ we multiply by the denominator 4 to change 2 into *fourths* and then divided by the 3 to find how many *three* fourths there are.

Divide 3 by $\frac{2}{8}$.

Why do you multiply by the denominator? Why do you divide by the numerator?

RULE FOR FRACTIONAL DIVISION WHERE ONLY THE DIVISOR IS A FRACTION.

Multiply the dividend by the denominator, and divide the product by the numerator.

EXAMPLES FOR THE SLATE.

Divide	23	by	$\frac{4}{7}$	Divide	364	by	$\frac{5}{9}$
"	25	"	$\frac{6}{12}$	"	24	"	$\frac{3}{4}$
"	32	"	$\frac{8}{12}$	"	21	"	$\frac{7}{8}$
"	325	"	$\frac{9}{15}$	"	486	"	$\frac{16}{42}$
"	9479	"	$\frac{24}{80}$	"	381	"	$\frac{18}{59}$
"	342	"	$\frac{16}{84}$	"	542	"	$\frac{34}{215}$
"	681	"	$\frac{15}{458}$	"	232	"	$\frac{42}{65}$
"	3292	"	$\frac{73}{516}$	"	4285	"	$\frac{12}{13}$

DIVISION WHERE THE DIVIDEND ONLY IS A FRACTION.

When the *dividend* only is a fraction, and we divide it by a *whole number*, we are to find how many *parts of a time*, a certain number is contained in *certain parts of a unit*.

Thus if we divide $\frac{1}{2}$ by 2, we know that $\frac{1}{2}$ does not contain 2

How is a number divided by a fraction? Why do we multiply 2 by the denominator 4? Why do we divide by the numerator 3? What is the rule for fractional division where only the divisor is a fraction?

units *once*, but we can find *what part of one time* the $\frac{1}{2}$ contains the 2.

If $\frac{1}{2}$ is divided by 1 *unit*, we find that it contains it, *not once*, but $\frac{1}{2}$ of once. It can contain *two* units but *half* as many times as *one* unit. Therefore $\frac{1}{2}$ contains 1 *one half* a time, and it contains 2 just half as often, or $\frac{1}{4}$ of a time. $\frac{1}{2}$ divided by 2 then is $\frac{1}{4}$. If $\frac{1}{2}$ is to be divided by 3, we reason in the same way. $\frac{1}{2}$ contains 1, $\frac{1}{2}$ a time. It contains 3 only a *third* as often. $\frac{1}{3}$ ot $\frac{1}{2}$ is $\frac{1}{6}$, and therefore $\frac{1}{2}$ contains 3, $\frac{1}{6}$ of a time.

Again, if $\frac{1}{2}$ is to be divided by 4, we reason thus:

If $\frac{1}{2}$ contains 1, $\frac{1}{2}$ a time, it contains 4 only $\frac{1}{4}$ as often. $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$. Then $\frac{1}{2}$ contains 4 not one time, but $\frac{1}{8}$ of one time.

Again let $\frac{1}{3}$ be divided by 4, and we reason thus:

If $\frac{1}{3}$ is divided by 1, it contains it not 1 time, but $\frac{1}{3}$ of one time. But it can contain 4 only $\frac{1}{4}$ as often. $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{12}$.

The dividend $\frac{1}{3}$ contains the divisor 4, not one time, but $\frac{1}{12}$ of one time.

Divide $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, each by *one*.

Proceed thus: $\frac{1}{5}$ contains 1 not *one* time, but $\frac{1}{5}$ of one time. $\frac{1}{6}$ contains 1 not one time, but $\frac{1}{6}$ of one time, &c.

Divide	$\frac{1}{5}$	by	2	Divide	$\frac{1}{6}$	by	3
"	$\frac{1}{6}$	"	4	"	$\frac{1}{7}$	"	5
"	$\frac{1}{8}$	"	6	"	$\frac{1}{9}$	"	10
"	$\frac{1}{12}$	"	7	"	$\frac{1}{10}$	"	7
"	$\frac{1}{10}$	"	8	"	$\frac{1}{12}$	"	9

How often is 2 contained in $\frac{1}{9}$?

Ans. As 1 would be contained $\frac{1}{9}$ of a time, 2 is contained *half* as often, or $\frac{1}{18}$ of one time.

How often is 3 contained in $\frac{1}{7}$?
How often is 5 contained in $\frac{1}{8}$?
How often is 6 contained in $\frac{1}{9}$?
How often is 7 contained in $\frac{1}{10}$?
How often is 8 contained in $\frac{1}{11}$?
How often is 9 contained in $\frac{1}{12}$?

When we divide a fraction by a whole number what do we find?

How often is 10 contained in $\frac{1}{2}$?
How often is 11 contained in $\frac{1}{3}$?
How often is 12 contained in $\frac{1}{4}$?
How often is 9 contained in $\frac{1}{5}$?
How often is 8 contained in $\frac{1}{7}$?
How often is 9 contained in $\frac{1}{4}$?

After finding how often 4 is contained in *one* part, we find by *multiplying*, how often it is contained in a *given number* of parts. For instance, 4 is contained in *one* fifth $\frac{1}{20}$ of *one time*. In *two* fifths it would be contained twice as often, or $\frac{2}{20}$ of one time.

Again, let $\frac{2}{7}$ be divided by 4, and we reason thus: 4 is contained in *one* seventh *one* fourteenth of one time, in 2 sevenths it is contained twice as often, or *two* fourteenths of one time.

How often is 3 contained in $\frac{4}{6}$?

Ans. 3 is contained in $\frac{1}{6}$ *one* eighteenth of one time. In $\frac{4}{6}$ it is contained 4 times as often, or *four* eighteenths of one time.

How often is 4 contained in $\frac{2}{7}$?
How often is 5 contained in $\frac{3}{8}$?
How often is 6 contained in $\frac{4}{9}$?
Divide $\frac{5}{10}$ by 3. Divide ^{3} by 5.
Divide $\frac{2}{9}$ by 6. Divide $\frac{4}{8}$ by 7. Divide $\frac{3}{12}$ by 8.
Divide $\frac{2}{5}$ by 9. Divide $\frac{5}{6}$ by 11. Divide $\frac{3}{4}$ by 8.
How many times is 6 contained in $\frac{3}{7}$?
How many times is 4 contained in $\frac{2}{9}$?
How many times is 7 contained in $\frac{3}{8}$?
How many times is 8 contained in $\frac{5}{9}$?
How many times is 9 contained in $\frac{3}{12}$?

In all the above cases it will be observed that the answer is obtained by simply *multiplying the denominator of the fraction by the divisor*.

Thus $\frac{2}{6}$ is divided by 4 thus. 4 is contained in $\frac{1}{6}$ $\frac{1}{24}$ of one time, and in $\frac{2}{6}$ *twice* as often, or $\frac{2}{24}$ of one time. It can be seen that the answer is obtained by multiplying the denominator of

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What method can *always* be taken in dividing a fraction by a whole number?

$\frac{2}{9}$ by the divisor 4. This is a method which can *always* be pursued in dividing any fraction by a whole number, viz: "multiply the *denominator* by the *divisor*."

But there is another method which is *sometimes* more convenient.

Let $\frac{8}{16}$ be divided by 4.

Now the quotient of 8 *units* divided by 4, is 2 *units*. Of course the quotient of 8 *sixteenths* divided by 4, is 2 *sixteenths*. In this case we have *divided the numerator* by the divisor 4. This can be done in all cases where the numerator can be divided *without remainder*.

But when a remainder would be left, it is best to divide, by *multiplying the denominator*. The answer is of the *same value* either way, though the name is different.

For example; in dividing $\frac{4}{9}$ by 2, we are to find how many times 2 is contained in $\frac{4}{9}$. Divide by *multiplying the denominator* by 2, and we find that it is contained not *once*, but $\frac{4}{18}$ of once. By *dividing the numerator* by 2, we find also that it is contained not once, but $\frac{2}{9}$ of once. Now $\frac{2}{9}$ and $\frac{4}{18}$ is the same value, by a different name. For if a thing is divided into *eighteen* parts, and we take *four* of them, we have the same value as if it were divided into *nine* parts and we took *two* of them.

Divide the following by both methods, and explain them as above.

Divide	$\frac{6}{8}$	by	3	Divide	$\frac{4}{12}$	by	2
"	$\frac{12}{16}$	"	4	"	$\frac{18}{20}$	"	6
"	$\frac{15}{17}$	"	5	"	$\frac{21}{84}$	"	7
"	$\frac{16}{50}$	"	8	"	$\frac{27}{85}$	"	9
"	$\frac{40}{80}$	"	10	"	$\frac{77}{100}$	"	11
"	$\frac{144}{250}$	"	12	"	$\frac{81}{600}$	"	9

RULE FOR DIVISION WHERE THE DIVIDEND IS A FRACTION.

Divide the numerator of the fraction by the divisor, or, (if this would leave a remainder,) multiply the denominator by the Divisor.

What other method is there? What is the rule for Division when the dividend only is a fraction?

EXAMPLES FOR THE SLATE.

In the following examples, *divide the numerator by the divisor.*

Divide	$\frac{12}{15}$	by	4	Divide	$\frac{132}{529}$	by	11
"	$\frac{25}{50}$	"	5	"	$\frac{468}{366}$	"	16
"	$\frac{32}{64}$	"	8	"	$\frac{56}{369}$	"	7
"	$\frac{50}{96}$	"	10	"	$\frac{150}{559}$	"	75
"	$\frac{144}{250}$	"	12				

In the following examples *multiply the denominator* by the divisor.

Divide	$\frac{3}{7}$	by	4	Divide	$\frac{8}{9}$	by	5
"	$\frac{5}{8}$	"	6	"	$\frac{6}{9}$	"	7
"	$\frac{6}{9}$	"	8	"	$\frac{7}{10}$	"	9
"	$\frac{8}{10}$	"	12	"	$\frac{9}{11}$	"	12
"	$\frac{15}{51}$	"	24	"	$\frac{72}{80}$	"	61

In the following examples *divide the numerator* by the divisor.

Divide	$\frac{30}{42}$	by	3	Divide	$\frac{24}{80}$	by	6
"	$\frac{32}{120}$	"	8	"	$\frac{36}{460}$	"	12
"	$\frac{64}{912}$	"	8	"	$\frac{81}{92}$	"	9
"	$\frac{49}{320}$	"	7	"	$\frac{42}{65}$	"	6
"	$\frac{28}{54}$	"	4	"	$\frac{36}{50}$	"	9

EXAMPLES FOR MENTAL EXERCISES.

1. If you have $\frac{4}{6}$ of an orange, and wish to divide it equally between two children, what part do you give each?

2. If you have $\frac{3}{4}$ of a load of hay, and divide it equally among 6 horses, how much do you give each?

3. If you have $\frac{3}{12}$ of a yard of muslin, and divide it into 3 equal parts, what part of a yard is each part?

4. If you have $\frac{16}{20}$ of an ounce of musk, and divide it into 12 equal portions, what part of an ounce is each portion?

5. If you divide $\frac{19}{20}$ of a dollar into 4 equal parts, what part of a dollar will each part be?

6. If a man owns $\frac{19}{30}$ of a cargo, and divides it equally among 4 sons, how much does he give each?

DIVISION OF ONE FRACTION BY ANOTHER.

When one fraction is to be divided by another, the same principle is employed, as when whole numbers are divided by a fraction.

For example, if the whole number 12 is to be divided by $\frac{3}{4}$, we first multiply by the denominator 4, to find how often *one* fourth is contained in 12, and then divide by 3, to find how often *three* fourths are contained in it.

In like manner, if we wish to find how many times, or *parts* of a time, $\frac{3}{4}$ is contained in $\frac{2}{12}$, we first find how often *one* fourth is contained in it, by reasoning thus :

One unit would be contained in $\frac{2}{12}$, two twelfths of one time.

One fourth would be contained *four* times as often, or $\frac{8}{12}$ of one time.

We thus find how often *one* fourth is contained in $\frac{2}{12}$, by multiplying it by 4, thus :

$$\frac{2}{12} \times 4 = \frac{8}{12}.$$

But *three* fourths would be contained only *one third* as often, and we find a third of $\frac{8}{12}$ by *multiplying its denominator* by 3. For when we wish to *divide a fraction* by 3, we multiply its denominator, and thus *make the parts* represented by the denominator, *three times smaller*, thus :

$$\frac{8}{12} \div 3 = \frac{8}{36}.$$

Here the *twelfths* are changed to *thirty-sixths ;* and a thirty-sixth is a *third* of one twelfth.

It will be found by examining the foregoing process, that in dividing one fraction by another, the fraction which is the *dividend* has its *numerator multiplied* by the *denominator of the divisor*, and its *denominator multiplied* by the *numerator of the divisor*.

Let another example be taken and observe thus.

Let $\frac{2}{3}$ be divided by $\frac{4}{5}$.

$\frac{2}{3}$ if divided by *one unit* would contain it *not once* but $\frac{2}{3}$ of once.

Is any different principle employed in dividing a fraction by a whole number? Explain the process. In the above example why was the numerator of the dividend multiplied by the denominator of the divisor?

But if divided by *one sixth* it would contain it 6 times as often or 6 times $\frac{2}{9}$, which is $\frac{12}{9}$.

Here the *numerator* of the dividend ($\frac{2}{9}$) has been multiplied by the denominator of the divisor ($\frac{4}{6}$), and we have thus found how often *one* sixth is contained.

Four sixths would be contained only *one fourth* as often, and we therefore divide $\frac{12}{9}$ by 4 by *multiplying its denominator* and the answer is $\frac{12}{36}$, and here the *denominator of the dividend* ($\frac{2}{9}$) has been multiplied by the *numerator of the divisor* ($\frac{4}{6}$).

We therefore multiplied the *numerator of the dividend* by the *denominator of the divisor* to find how often *one* sixth was contained, and multiplied the *denominator of the dividend* by the *numerator of the divisor* to find how often *four* sixths were contained.

Let the following be performed and explained as above.

Divide	$\frac{2}{4}$	by	$\frac{5}{6}$	Divide	$\frac{4}{8}$	by	$\frac{3}{7}$
"	$\frac{3}{8}$	"	$\frac{6}{8}$	"	$\frac{2}{12}$	"	$\frac{9}{10}$
"	$\frac{3}{12}$	"	$\frac{5}{9}$	"	$\frac{13}{20}$	"	$\frac{6}{13}$

This process corresponds with that used in dividing a *whole number by a fraction.*

For if we divide 12 by $\frac{3}{4}$ we first multiply it by 4 to find how many *one fourths* there are in 12, and then divide the answer by 3 to find how many *three* fourths there are.

So in dividing $\frac{2}{6}$ by $\frac{3}{4}$ we first multiply it by 4 to find how many times *one fourth* is contained thus ($\frac{8}{6}$), and then divide it by 3 to find how many times *three fourths* are contained thus, ($\frac{8}{18}$).

Examples.

Divide	$\frac{2}{3}$	by	$\frac{4}{5}$	Divide	$\frac{3}{8}$	by	$\frac{2}{12}$
"	$\frac{5}{10}$	"	$\frac{6}{9}$	"	$\frac{3}{12}$	"	$\frac{4}{6}$
"	$\frac{6}{12}$	"	$\frac{9}{12}$	"	$\frac{6}{8}$	"	$\frac{5}{9}$

Why was the denominator of the dividend multiplied by the numerator of the divisor? Explain how this process corresponds with that used in dividing whole numbers.

We *invert* a fraction when we *exchange the places of the numerator and the denominator.*

Thus $\frac{1}{2}$ inverted is $\frac{2}{1}$, and $\frac{3}{6}$ inverted is $\frac{6}{3}$, and $\frac{13}{50}$ inverted is $\frac{50}{13}$, &c.

Now it appears, as above, that if we wish to divide $\frac{3}{4}$ by $\frac{2}{6}$ we are to multiply its numerator (3) by the denominator (6) and its denominator (4) by the numerator (2). This is more easily done, if we *invert the divisor* $\frac{2}{6}$, thus $\frac{6}{2}$.

When the divisor is thus inverted we can *multiply the numerators together* for a new numerator and the *denominators for a new denominator* and the process is the same.

Thus let us divide $\frac{4}{6}$ by $\frac{2}{3}$.

Inverting the divisor $\frac{2}{3}$ the two fractions would stand together thus $\frac{4}{6}$ $\frac{3}{2}$. We now multiply the numerators and denominators together and the answer is $\frac{13}{12}$, and it is the same process, as if we had not inverted the divisor, but multiplied the numerator of the dividend by the denominator of the divisor and its denominator by the numerator of the divisor.

This method therefore is given as the easiest rule, but it must be remembered that in this process we always *multiply the dividend* by the *denominator of the divisor* and *divide it* by the *numerator,* as we do in case of whole numbers.

COMMON RULE FOR DIVIDING ONE FRACTION BY ANOTHER.

Invert the divisor, and then multiply the numerators and denominators together.

EXAMPLES FOR THE SLATE.

Divide $\frac{26}{28}$ by $\frac{7}{12}$.

Invert the divisor and the fractions stand thus $\frac{26}{28}$ $\frac{12}{7}$.

Multiply them together, and the answer is $\frac{312}{196}$.

Divide	$\frac{87}{90}$	by	$\frac{54}{75}$	Divide	$\frac{18}{19}$	by	$\frac{72}{85}$
"	$\frac{32}{49}$	"	$\frac{56}{89}$	"	$\frac{14}{19}$	"	$\frac{14}{11}$
"	$\frac{38}{56}$	"	$\frac{32}{21}$	"	$\frac{56}{412}$	"	$\frac{92}{506}$
"	$\frac{65}{138}$	"	$\frac{341}{362}$	"	$\frac{16}{49}$	"	$\frac{92}{102}$

How is a fraction inverted? What is the common rule for dividing one fraction by another?

DECIMAL ADDITION.

RULE FOR ADDING DECIMALS.

Place figures of the same order under each other. Add each column, as in Simple Addition, and in the answer place a separatrix between the orders of units and tenths.

EXAMPLE.

What is the sum of 234,406? 4,6490? 13,234? 2,2? 3650, 4002? 999,4699?

Placing units of the same order under each other, they stand thus:

```
 234,406
   4,6490
  13,234
   2,2
3650,4002
 999,4699
---------
4904,3591
```

Let the pupils proceed as in Simple Addition, calling the names of each order, thus :—

9 tenths of thousandths added to 2, are 11 tenths of thousandths; which is 1 *tenth of thousandths*, to be written under that order; and one of the order of *thousandths*, to be carried to that order.

1 thousandth carried to 9, is 10, and 4 are 14, and 9 are 23, and 6 are 29 thousandths; which is 9 *thousandths*, to be written under that order, and 2 *hundredths*, to be carried to the next order.

Thus through the other orders, observing to *place a separatrix between the orders of units and tenths.*

Arrange the following mixed decimals according to their orders, and then add them.

(1)

306,42001. 20,3391. 3246,42. 39,4695. 634,001. 84,6302.

(2)

99, 987. 65432,02564. 64,65. 596,32. 87632,51739. 36,50. 51639,2154.

(3)

63,204. 6359,42591. 8642,39. 86423,2915. 68,241.

(4)

63,9876. 59432,1103. 95,02. 876,3254. 8634,251. 3426,549.

What is the rule for Decimal Addition?

EXERCISES FOR OLDER PUPILS.

Let the pupil write and add the following sums in Decimals, *remembering to place units of the same order under each other.*

1.

Four units, six tenths, four hundredths, five thousandths.
Two tens, four units, six hundredths.
Three tens, two units, two hundredths, seven thousandths.
Six units, five tenths, seven hundredths, four thousandths, three tenths of thousandths.
One unit, three tenths.

2.

Forty-two units; sixteen *thousandths.*
Five units; sixty-three *hundredths of thousandths.*
Seventy-four units; seven thousand five hundred and fifty-three *tenths of thousandths.*
Two units; five hundred and sixty *tenths of thousandths.*

3.

Two hundred and forty-three units; two hundred and forty-three *thousandths.*
Seventeen units, nine hundred and seventy-three *tenths of thousandths.*
Fifty units; six thousand seven hundred and forty-three *hundredths of thousandths.*
Five units; eight *thousandths.*
One thousand units; one thousand *tenths of thousandths.*

4.

One thousand and one units; one thousand and one *hundredths of thousandths.*
Nine hundred and ninety-nine units; nine thousand nine hundred and thirty *hundredths of thousandths.*
Four units; thirty *tenths of thousandths.*
Five units; fifty-five thousand and forty-three *millionths.*

5.

Sixteen units; seven hundred and sixty-four *thousandths.*
Two units; forty-five *hundredths of thousandths.*
Fifty units; forty-two *millionths.*
Seven units; nine hundred and ninety-eight *tenths of thousandths.*
Six units; five hundred and forty-nine *millionths.*

6.

Four thousand units; four thousand *tenths of thousandths.*
Forty-one units; four thousand, four hundred and nine *hundredths of thousandths.*
Seven units; eighty-seven *tenths of thousandths.*
Four hundred and forty-one units; ninety-nine *hundredths.*
Four units, four *hundredths of thousandths.*

7.

Seventeen units; nine thousand eight hundred and sixty *hundredths of thousandths.*
Nine units; sixteen *tenths of thousandths.*
Four units; fifty-five *hundredths.*

Sixty-three units; ninety-nine *millionths.*
One unit; seventy-four *thousandths.*

8.

Five hundred and forty-four units; eight thousand seven hundred and fifty-five *millionths.*
Ninety-nine units; four hundred *hundredths of thousandths.*
Six units; eight hundred and eighty-eight *thousandths.*
Eight thousand units; seventy-four *tenths of thousandths.*
Six units; eighty-eight *hundredths,*

9.

Seventeen units; forty *thousandths.*
Five units; ninety-three *millionths.*
Forty-four units; eighty-seven *hundredths.*
Six units; nine hundred and ninety-nine *thousandths.*
Four hundred and twelve units; seventy-five *tenths of thousandths.*

10.

Seventy-eight units; four thousand and five *tenths of thousandths.*
Two units; five hundred *hundredths of thousandths.*
Seven units; eighty-nine *millionths.*
Five hundred and seventy-two units; seventy-six thousand, eight hundred and sixty-four *hundredths of thousandths.*
Nine thousand and fifty units; nine thousand and fifty *millionths.*

11.

Five hundred and eighty-seven units; twenty-nine hundred *tenths of thousandths.*
Forty units; five hundred and sixteen *millionths.*
Eight units; four hundred and ninety-six thousand *millionths.*
Five hundred and forty-two units; two thousand *hundredths of thousandths.*
Seventeen units; nine thousand nine hundred *hundredths of thousandths.*

12.

Sixty-five units; sixty-five *hundredths of thousandths.*
One hundred and eighty units, one hundred and eighty *tenths of thousandths.*
Twenty-four units; twenty-four *millionths.*
Sixteen units; sixteen *hundredths.*
Five units; five *thousandths.*
Fifty units; fifty *hundredths of thousandths.*

DECIMAL SUBTRACTION.

If 2 tenths, 4 hundredths of a dollar, be taken from 4 tenths, 6 hundredths, what will remain?

If 3 hundredths, 5 thousandths of a dollar, be taken from 5 hundredths, 7 thousandths, what will remain?

If 5 dimes, 6 mills, be taken from 7 dimes, 8 mills, how much will remain?

If 4 dimes, 5 cents, be taken from 7 dimes, 9 cents, how much will remain?

If 4 units, 6 tenths, be taken from 6 units, 8 tenths, how much will remain?

In simple subtraction, if the number in any order of the minuend, was smaller than the one to be subtracted, what did you do?

The same is to be done in Decimal Subtraction.

Take 4 tenths, 7 hundredths of a dollar, from 6 tenths, 5 hundredths.

In which order is the number of the subtrahend the largest?

Can 7 hundredths be taken from 5 hundredths? What must be done in this case?

Take 5 dimes, 6 cents, from 8 dimes, 9 cents.

In which order is the number of the subtrahend the largest?

Can 9 cents be taken from 6 cents? What must you do in order to subtract?

Subtract 7 hundredths, 8 thousandths of a dollar, from 8 hundredths, 7 thousandths.

Can 8 thousandths be subtracted from 7 thousandths? What must be done in this case?

RULE FOR DECIMAL SUBTRACTION.

Proceed by the rule for common Subtraction, and in the answer place a separatrix between the orders of units and tenths.

EXAMPLE.

Subtract 2,56 from 24,329. Placing the subtrahend under the minuend, so that units of the same order stand in the same column. They stand thus:

```
 24,329
  2,56
 ------
 21,769
```

Let the pupil learn to subtract in this manner:

Nothing from 9 thousandths, and 9 remains to be set down. 6 hundredths cannot be taken from 2 hundredths; we therefore add 10 hundredths to the minuend, which makes 12. 6 taken from 12 leaves 6. As 10 was added to the minuend, an equal quantity must be added to the subtrahend. 1 of the order of tenths is the same as 10 hundredths, we therefore add 1 to the 5 tenths, making it 6 tenths. 6 tenths cannot be taken from 3

What is the rule for decimal subtraction? Employ the method above in adding other decimals.

tenths, we therefore add 10 tenths to the minuend, which makes 13. 6 taken from 13, leaves 7. As 10 tenths was added to the minuend, an equal amount must be added to the subtrahend. 1 of the order of units is the same as 10 tenths, we therefore add 1 to the 2 units, making it 3 units.

Proceed thus through all the orders, remembering to place a separatrix between the orders of units and tenths.

Let the following sums be arranged and subtracted in the same way:

Subtract	25,25	from	62,904
"	790,4	"	996,409
"	2,4693	"	354,268
"	5,34689	"	40,62
"	6,6543	"	23,3291
"	432,54916	"	542,65329
"	53,00300	"	646,01201
"	832,2	"	9988,659
"	51,895	"	64,59432
"	8,4156	"	400,21
"	321,01013	"	4333,0063
"	659,09543	"	679,2941

Exercises for older pupils.

1.

Subtract two tens, four units, three tenths, five hundredths, and four thousandths; from four tens, two tenths, five hundredths, and four thousandths.

2.

Subtract two tens, three units, six tenths, nine hundredths, and three thousandths; from four tens, four units, three thousandths, and five tenths of thousandths.

3.

Subtract two units; four thousand three hundred and seventy-four *tenths of thousandths*; from

Twenty-three units; seven thousand five hundred *tenths of thousandths.*

4.

Subtract ninety-eight units, two thousand nine hundred and eighty-seven *tenths of thousandths*; from

Seven hundred and seventy-seven units, four thousand three hundred and twenty-six *tenths of thousandths.*

5.

Subtract seven units, six thousand five hundred and forty-three *tenths of thousandths*; from

Three hundred and sixty-nine units, forty-two *hundredths.*

6.

Subtract seventy-seven units, twenty-four *tenths of thousandths*; from Two hundred and twenty-five units, seven thousand six hundred and fifty-four *tenths of thousandths.*

7.

Subtract twelve units, one *millionth*; from thirty units, ten *thousandths.*

8.

Subtract one hundred units, eleven *tenths of thousandths*; from Three hundred units, one *tenth.*

9.

Subtract five hundred and fifty *millionths*; from ninety five *hundredths.*

10.

Subtract ninety-eight units, fifty-four *tenths of thousandths*; from Eight hundred and eighty-seven units, thirty-four *tenths of thousandths.*

11.

Subtract twenty units, seven thousand three hundred and twenty-one *tenths of thousandths*; from

Thirty-nine units, eighty-four thousand, three hundred and twenty-one *hundredths of thousandths.*

12.

Subtract forty units, twenty-five thousands, nine hundred and eighty-three *hundredths of thousandths*; from

Eight hundred and forty-one units, six hundred and forty-three *tenths of thousandths.*

13.

Subtract eight units, forty-one *tenths of thousandths*; from Seventy-seven units, forty-three thousand and eleven *millionths.*

14.

Subtract eight units, one thousand and fourteen *millionths*; from Eight hundred units, twenty-one *tenths of thousandths.*

15.

Subtract four hundred units, sixty *hundredths*; from One thousand units, three *tenths.*

16.

Subtract fifteen hundred *millionths*; from Eighteen *hundredths of thousandths.*

17.

Subtract eighty units, eighty *thousandths*; from Eight hundred units, and eighty *millionths.*

18.

Subtract two units, seventy-six thousand and eight *millionths*; from Nine hundred and eighty-seven units, forty-four *hundredths of thousandths.*

DECIMAL MULTIPLICATION.

In explaining decimal multiplication, it is needful to understand the mode of multiplying and dividing by the *separatrix.*

If we have 2,34 we can make it *ten* times greater, by moving the separatrix one order to the right, thus, 23,4. For 23 units, 4 tenths, is *ten* times as much as 2 units, 34 hundredths.

It is therefore multiplied by 10.

We can multiply it by 100 by removing the separatrix entirely, thus, 234, for the two units and 34 hundredths, become 234 units, and are thus multiplied by 100.

Whenever therefore we wish to multiply a mixed or pure decimal, by any number composed of 1 and ciphers, we can do it by moving the separatrix *as many orders to the right*, as there are *ciphers* in the *multiplier*.

EXAMPLES.

Multiply	462,5946	by	100
"	2,6395	"	1000
"	4,63956	"	10000
"	54,6329	"	10
"	4,6930	"	1000
"	,3694	"	100
"	4,6934	"	10000

But if the decimal has not as many *figures* at the right, as are needful in moving the separatrix, *ciphers* can be added thus. Multiply 2,5 by 1000. Then in order to multiply by a number composed of one and ciphers, it is necessary to move the separatrix as many orders to the right, as there are ciphers in the multiplier, 1000; in order to do this, two ciphers must be added thus,

2500,

Here 2 *units*, and 5 *tenths*, are changed to 2 *thousands* and 5 *hundreds*, and of course are made 1000 times larger, or multiplied by 1000.

In the following examples, in order to multiply by *moving the separatrix*, it is necessary to add ciphers to the right of the multiplicand.

EXAMPLES.

Multiply	3,7	by	100	Multiply	5,2	by	100
"	2,35	"	1000	"	36,3	"	1000
"	2,5	"	10000	"	3,869	"	10000
"	34,200	"	100000	"	5,6469	"	100000

How can decimals be multiplied by any number composed of 10 ciphers? What is done if the decimal has not as many figures at the right as are required?

Division also, can be performed on decimals, by the use of the separatrix.

Whenever we divide a number, we make it as much *smaller*, as the divisor is *greater than one.*

If we divide by 10, as 10 is ten times greater than *one*, we make the number 10 times *smaller*.

If we divide by 100, we make the numbers 100 times smaller.

If therefore we make a number 10 or 100 times smaller, we divide by 10 or 100.

If we make it 1000 times smaller, we *divide* by 1000, &c.

If then we are to divide 323,4 by 10, we must make it 10 times smaller. This we can do by moving the separatrix one order to the left, thus, 32,34. If we are to divide by 100, we can do it by moving the separatrix *two* orders to the left, thus, 3,234.

If we are to divide by 10000, we can do it by moving the separatrix 4 orders to the left, thus, ,3234.

Whenever therefore, we wish to divide a pure or mixed decimal, by a number composed of 1 and ciphers, we can do it by moving the separatrix as *many orders* to the *left*, as there are *ciphers* in the *divisor*.

EXAMPLES.

Divide	32,5	by	10	Divide	32,69	by	10
"	342,6	"	100	"	3269,1	"	100
"	469,3	"	1000	"	2396,4	"	1000
"	46936,7	"	10000	"	12346,95	"	10000
"	23469,8	"	100000	"	15463,96	"	100000

But if the decimal has not enough figures to enable the separatrix to be moved, according to the rule, ciphers must be *prefixed*.

Thus if we wish to divide 3,2 by 100, we do it thus, ,032. Here the 3 is changed from 3 *units*, to 3 *hundredths*, and of course made 100 times less.

EXAMPLES.

Divide	2,4	by	100	Divide	23,4	by	10000
"	32,4	"	1000	"	246,4	"	100000
"	932,5	"	10000	"	293,6	"	100000

When we divide a number, how much smaller do we make it? How can we divide a decimal by any number composed of 1 and ciphers? What is done if the decimal has not figures enough?

Divide	21,6	by	100000	Divide	546,9	by	100000
"	600,7	"	1000000	"	32,3	"	1000000
"	286,9	"	10000000	"	100,4	"	10000000
"	542,8	"	100000000	"	3694,9	"	1000000

A decimal can also be multiplied, by *expunging* the separatrix.

Thus 2,4 is multiplied by 10, by expunging the separatrix, thus, 24.

2,56 is multiplied by 100, by expunging the separatrix, thus, 256.

In all these cases, the decimal is multiplied by a number composed of 1, and *as many ciphers as there are decimals at the right* of the separatrix which is expunged.

If you expunge the separatrix of the following decimals, by what number are they multiplied?

2,46.	3,295.	54,6823.
54,63.	89,46321.	5,6432.

How can you multiply 3,1 by 10? What is it after this multiplication?

How do you multiply 3,12 by 100? What is it after this multiplication?

How do you multiply 9,567 by 1000? What is it after this multiplication?

If the separatrix is expunged from 2,52, by what is it multiplied?

If the separatrix is expunged from 2,56934, by what is it multiplied?

If the separatrix is removed from 5,943216, by what is it multiplied?

If the separatrix is removed from 3,4621, by what is it multiplied?

If the separatrix is removed from 3,5, by what is lt multiplied?

If a man supposes he owes $54,23, and finds he owes 10 times as much, what is the sum he owes? How do you perform the multiplication with the separatrix? What does the number become after being thus multiplied?

Multiply in the above mode $244,635 by 10, by 100, and by 1000. What does the sum become, by each of these operations?

Divide $244,635 by 10, by100, and by 1000, with the separatrix. What does the sum become by each of these operations?

What effect is produced by expunging the separatrix of a decimal? In this case by what number is the decimal multiplied?

Divide and multiply, with a separatrix, $2556,436, by 10, by 100, and 1000.

If before *multiplying*, the *multiplicand* is made a certain number of times *larger*, the product is made *as much larger*. If the multiplicand is made *too* large, the product is *as much too large*.

For example;

If we wish to find how much *twice* 2,3 is, we can change it to whole numbers, and multiply it by 2, and we know the answer is 10 times too large. For 23 is 10 times larger than 2,3, and therefore when it is multiplied by 2, its product is 10 times too large. If then we make it 10 times smaller, we shall have the right answer. Whenever, therefore, we wish to multiply a decimal, we can change it to whole numbers, and multiply it by the rule for common multiplication. We then can make the product as much smaller, as we made the multiplicand larger, by changing it to whole numbers.

For instance, if we wish to multiply 3,6 by 3, we can expunge the separatrix, and the multiplicand becomes 10 times too large. We then multiply it as in whole numbers thus,

$$\begin{array}{r} 36 \\ 3 \\ \hline 108 \end{array}$$

This *product* is also 10 times too large, and we find the right answer, by placing a separatrix so as to *divide* it by 10, thus making it ten times smaller.

In like manner, if the *multiplier* is increased a certain number of times, the *product* is increased *in the same proportion*.

If we are to multiply 32 by 2,3, and should by expunging the separatrix, change the multiplier to whole numbers, it would make the product 10 times too large, and to obtain the right answer we must divide the product by 10 with a separatrix, thus making it 10 times smaller.

Multiply 2,5 by 4.

By what number do you multiply, when you expunge the separatrix of the decimal?

What is the product of the multiplication after the separatrix is expunged? How much too large is this product?

How do you divide this product by the same number as you multiplied the decimal?

Explain each process as above.

What is the effect on the product, if the multiplicand is made a certain number of times larger? How is the right product to be obtained? What is the effect on the product, if the multiplier is increased a certain number of times?

Multiply	12,46	by	5	Multiply	3,2	by	6
"	18,23	"	8	"	52,23	"	7
"	,346	"	9	"	286,45	"	8
"	36,2	"	7	"	123,678	"	9
"	25,36	"	5	"	32,92	"	12
"	44,429	"	4	"	64,64	"	11
"	92,1234	"	7	"	988,931	"	9

Multiply	329	by	2,4	Multiply	764	by	8,925
"	426	"	3,5	"	2875	"	72,63
"	362	"	39,5	"	30021	"	984,4
"	4689	"	2,36	"	8643	"	6,529
"	4693	"	5,462	"	2875	"	,462
"	32678	"	6,8246	"	7628	"	,3596

Let the multiplier be 2,4, and the multiplicand is 3,6. Changing the *multiplier* to whole numbers, would make the product *ten* times too large. Should the *multiplicand* be changed to whole numbers, the product would *again* be made ten times larger, so that it would be made 100 times too large. Therefore to bring the answer right, we must divide it by 100, thus making it 100 times smaller. This is done by the use of a separatrix. 3,6 and 2,4, when changed to whole numbers and multiplied together, are 864. This is 100 times too large, and is brought right, by dividing it by 100, thus, 8,64.

RULE FOR EXPLAINING DECIMAL MULTIPLICATION.

Change the Decimals to whole numbers by expunging the separatrix. Multiply as in whole numbers. Divide the answer by the product of the two numbers by which the factors were multiplied, in expunging the separatrix.

EXAMPLE

Multiply 8,61 by 4,7.

Change these to whole numbers, and they become 861 and 47. (Here the multiplicand, in expunging the separatrix, is multiplied by 100, and the multiplier by 10.) Multiplying them together, they produce 40467. The *product* of the two numbers by which the factors were multiplied, (10 and 100), is 1000. Dividing 40467 by it, gives the answer 40,467.

How is the right product obtained? What is the rule for *explaining* the process of decimal multiplication?

EXAMPLES.

Multiply 2,37 by 4,6.

By what do you multiply each factor when you remove the separatrix? What is the product of the two numbers by which you multiplied the factors?

How do you divide by this product?

Multiply	2,64	by	3,8
"	362,68	"	48,72
"	6895,40	"	3,651
"	334,02	"	28,54
"	2195,334	"	3,2
"	3456,567	"	,51
"	937,8	"	,84
"	1234,636	"	36,4
"	765,3	"	1,23
"	89123,002	"	,591

The following common rule for decimal multiplication, includes all the others, and may be used after understanding the preceding.

COMMON RULE FOR DECIMAL MULTIPLICATION.

Multiply as in whole numbers, and then point off in the product, as many orders of decimals, as are found in both the factors.

EXAMPLES.

Multiply	3,69	by	3,8	Multiply	12	by	4,6
"	18,600	"	5,9	"	1,94	"	,600
"	224,7	"	2,3	"	351,9	"	6
"	9,427	"	3,4	"	,658	"	,236

DECIMAL DIVISION.

In order to understand the process of Decimal Division, it is needful to recollect the method of dividing and multiplying, by ciphers and a separatrix.

If we wish to multiply a number by a sum composed of 1 *with ciphers added to it*, we add as many ciphers to the multiplicand, as there are ciphers in the multiplier. Thus if we wish to multiply 64 by 10, we do it by adding *one* cipher, 640. If we are to multiply by 100, we add two ciphers thus, 6400, &c.

EXAMPLES.

Multiply	3	by	100	Multiply	46	by	100
"	19	"	1000	"	2	"	100000

If we wish to multiply a *decimal* by any number composed of 1 with ciphers annexed, we can do it by *removing the separatrix as many orders to the right, as there are ciphers in the multiplier.*

Thus if ,2694 is to be multiplied by 10, we do it thus; 2,694. If it is to be multiplied by 100, we do it thus; 26,94. If it is to be multiplied by 1000 we do it thus; 269,4. But to multiply by *a million*, we must *add ciphers* also, in order to be able to move the separatrix as far as required, thus; 269400,.

EXAMPLES.

Multiply	2,64	by	10	Multiply	6,4	by	10000
"	36,9468	"	100	"	1,643	"	10
"	3,2	"	1000	"	3,2	"	1000000

The same method can be employed in *dividing* decimals, by any number composed of 1 and ciphers annexed.

The rule is this. *Remove the separatrix as many orders to the left, as there are ciphers in the divisor.*

Thus if we wish to divide 23,4 by 10, we do it thus; 2,34.

If we wish to divide it by 100 we do it thus, ,234. But if we wish to divide it by a thousand it is necessary to *prefix* a cipher thus, ,0234. If we divide it by 10,000 we do it thus, ,00234.

EXAMPLES.

Divide	2,4	by	100	Divide	24,3	by	10
"	2,46	"	10	"	246,9	"	100
"	3,2	"	1000	"	2,3	"	100000
"	2,4	"	10	"	34,26	"	1000
Multiply	2,4	"	10	Mult'y.	34,26	"	1000
Divide	328,94	"	100	Divide	3,2	"	10000
Multiply	326,94	"	100	Multiply	3,2	"	10000

It is needful to understand, that a mixed decimal can be changed to an *improper decimal fraction*.

For example, if we change 3,20 to an improper decimal fraction, it becomes 320 *hundredths* ($\frac{320}{100}$), which is an improper fraction, because its numerator is larger than the denominator.

What is the rule for dividing decimals by any number composed of 1 and ciphers? What can a mixed decimal be changed to? Give an example.

But we cannot express the denominator of 320 hundredths, by a separatrix in the *usual* manner, for the rule requires the separatrix to stand, so that there will be as many figures at the right of it, as there are ciphers in the denominator.

If then we attempt to write 320 hundredths in this way, it will stand thus, 3,20, which is then a *mixed decimal* and must be read *three units* and 20 hundredths. If it is written thus, $\frac{320}{100}$ it is then a *vulgar* and not a decimal fraction.

But it is convenient in explaining several processes in fractions, to have a method for expressing *improper decimal fractions*, without writing their denominator. The following method therefore will be used.

Let the *inverted separatrix* be used to express an *improper* decimal fraction. Thus let the *mixed decimal* 2,4 which is read *two* and *four tenths*, be changed to an *improper decimal* thus, 2‘4 which may be read *twenty-four tenths*.

The denominator of an improper decimal, (like that of other decimals) *is always* 1 and *as many ciphers as there are figures at the right of the separatrix*. It is known to be an *improper* decimal, simply by having its separatrix inverted.

Thus 24‘69 is read, two thousand four hundred and sixty-nine *hundredths*. 239‘6 is read, two thousand three hundred and ninety-six *tenths*, &c.

EXAMPLES.

Change the following *mixed decimals* to *improper decimals*, and read them.

246,3	24,96	32,1
326,842	3,6496	49,2643
8,4692	368,491	26,3496

RULE FOR WRITING AN IMPROPER DECIMAL.

Write as if the numerator were whole numbers, and place an inverted separatrix, so that there will be as many figures at the right, as there are ciphers in the denominator.

Write the following *improper decimals*.

Three hundred and six *tenths*.

Four thousand and nine *hundredths*.

Two hundred and forty-six thousand, four hundred and six *tenths*.

What is the denominator of an improper decimal? How is it known to be an improper decimal? What is the rule for writing improper decimals?

Three millions, five hundred and forty-nine *tenths of thousandths.*

Two hundred and sixty-four thousand, five hundred and six *thousandths.*

Five hundred and ninety-six *tenths.*

DECIMAL DIVISION WHEN THE DIVISOR IS A WHOLE NUMBER.

The rules for Decimal Division are constructed upon this principle, that any quotient figure must always be put in the same order as the *lowest order* of that part of the dividend taken.

Thus if we divide ,25 (or two tenths, five hundredths,) by 5, the quotient figure must be put in the *hundredth* order, thus, (,05) because the lowest order of the dividend is hundredths.

Again, if ,250 is divided by 50, the quotient figure must be 5 *thousandths*, (,005) for the same reason.

Let us then divide ,256 by 2. We proceed exactly as in the Short Division of whole numbers, except in the use of a *separatrix.* Let the pupil proceed thus:

```
2),256
  ,128
```

2 tenths divided by 2, gives 1 as quotient, which is 1 *tenth*, and is set under that order with a separatrix before it. 5 hundredths divided by 2, gives 2 as quotient, which is 2 *hundredths*, and is set under that order.

1 hundredth remains, which is changed to *thousandths*, and added to the 6, making 16 *thousandths.*

This, divided by 2, gives 8 *thousandths* as quotient, which is placed in that order.

If the divisor is a whole number, and has *several orders* in it, we proceed as in Long Division, except we use a separatrix, to keep the figures in their proper order. Thus if we divide 15,12 by 36, we proceed thus:

We first take the 15,1, and divide it, remembering that the quotient figure is to be of the same order as the *lowest order* in the *part of the dividend taken*, of course the quotient 4 is 4 *tenths* (,4) and must be written thus in the quotient.

```
36)15,12(,42
   14,4
   ----
     ,72
     ,72
     ---
     ,00
```

On what principle are the rules for decimal division constructed? Explain the example given. If the divisor is a whole number, and has several orders, how do we proceed?

We now subtract 36 times ,4 which is 14,4, (see rule for Decimal Multiplication, page 132) from the part of the dividend taken and 7 *tenths* (,7) remain.

To this bring down the 2 *hundredths*. Divide, and the quotient figure is 2 *hundredths*, which must be set in that order in the quotient.

Subtract 36 times ,02 (or ,72) from the dividend and nothing remains.

Let the following sums be performed and explained as above.

Divide	76,8	by	24	Divide	37,8	by	21
"	94,6	"	43	"	85,8	"	26

Sometimes ciphers must be prefixed to the first quotient figure, to make it stand in its proper order.

For example, let ,1512 be divided by 36, and we proceed thus,

```
36),1512(,0042
    ,144
    -----
    ,0072
    ,0072
    -----
     0000
```

We take ,151 first, which is 151 *thousandths* (for the *denominator* of any decimal is always of the same order as the *lowest order* taken).

This divided by 36 gives 4 as quotient. This 4 is 4 *thousandths*, because the *lowest order* in the part of the dividend taken is thousandths. Therefore when it is put in the quotient it must have *two ciphers and a separatrix prefixed* thus ,004.

We now subtract from the dividend 36 times, ,004 or ,144. (See rule for Decimal Multiplication.)

It is desirable in such cases to place ciphers and a separatrix in the *remainder*, to make them stand in their proper orders.

To the remainder (,007) bring down the 2 *tenths of thousandths*, making 72 *tenths of thousandths*.

This divided by 36 gives 2 *tenths of thousandths* as quotient, which is set in that order. 36 times 2 tenths of thousandths (or ,0072) being subtracted, nothing remains.

Sometimes we must add ciphers to the dividend before we can begin to divide.

For example, let ,369 be divided by 469, and we proceed thus,

---

Explain the example given. Of what order is the denominator of any decimal?

	469),3690(,00078
	,3283
	,04070
	,03752
	,00318 $\frac{318}{469}$

We find that ,369 cannot be divided by 469, so we add a cipher to it, making it 3690 *tenths of thousandths.*

This divided by 469 gives 7 as quotient, which is 7 *tenths of thousandths,* (,0007) because the lowest order of the dividend is of that order.

We now subtract 469 times ,0007 (which is ,3283) from the dividend, and ,0407 remain.

To this remainder we add a cipher, and change it from 407 *tenths of thousandths* to 4070 *hundredths of thousandths.*

This divided by 469 gives 8 as quotient, which is 8 *hundredths of thousandths,* because the lowest order in the dividend is hundredths of thousandths.

We now subtract 469 times 8 hundredths of thousandths (or ,03752) from the dividend and ,00318 remain.

We could continue dividing, by adding ciphers to the remainders, but it is needless. Instead of this we can set the divisor under the remainder as in common division, thus $\frac{318}{468}$.

It is not needful to retain the separatrix and ciphers when thus writing a remainder, because when put in the quotient, it is not considered as the $\frac{318}{468}$ *part* of a whole number, but as a part of the *lowest order in the decimal, by which it is placed.*

Thus when this is put with the above quotient, we read the answer thus, 78 *hundredths of thousandths,* and $\frac{318}{469}$ of *another hundredth of thousandth.*

Let the following sums be performed and explained as above.

Divide	3,694	by	84	Divide	42869	by	95
"	,36946	"	841	"	3,69428	"	49
"	3,26	"	589	"	,260	"	482
"	32,4	"	386	"	481,4	"	81
"	364,6	"	99	"	28,1	"	15

Decimal Division when the Divisor is a Decimal.

When the *divisor* is a *decimal,* we proceed as in dividing by a *Vulgar Fraction,* viz.

We *multiply by the denominator, and divide by the numerator.*

Thus if we are to divide 24 by ,4, we are to find how many 4 tenths there are in 24.

What is the rule for decimal division, when the divisor is a decimal?

We first multiply 24 by the denominator 10, to find how many *one* tenths there are, and then divide by the numerator 4, to find how many 4 tenths there are. 24 is multiplied by ten, thus; 24‘0, and has the inverted separatrix, to show that it is not 240 *whole numbers*, but *tenths*.

We now have found that in 24 there are 240 *one* tenths, we now divide by 4, to find how many 4 tenths there are. The answer is 60, which according to the rule, must be of the same order as the *lowest order* in the dividend, or 60 *tenths*, and must be shown by the inverted separatrix thus (6‘0.) This may be changed to whole numbers by *reverting* the separatrix thus (6,0.)

When the dividend is a *decimal*, we can multiply by removing the separatrix.

Thus let 8,64 be divided by ,36.

Here we are to multiply by 100, to find how many *one hundredths* there are in the dividend, and then divide by 36 to find how many 36 hundredths there are.

We multiply by 100, by removing the separatrix two orders toward the *right*, and then dividing by 36, we have 24 as answer, which is 24 units, because the dividend is units.

```
36)864,(24
   72
   ---
   144
   144
   ---
   000
```

If the divisor is a *mixed decimal*, we change it to an *improper* decimal, and then proceed as before, *multiplying by the denominator and dividing by the numerator*.

Thus let 10,58 be divided by 4,6.

We first change the divisor into an improper decimal thus, 4‘6 (46 *tenths*.)

We now are to multiply the 10,58 by 10, to find how many *one* tenths there are, and then divide by 46, to find how many 46 tenths there are.

We multiply by 10 by removing the separatrix thus, 105,8, and proceed as follows.

```
46)105,8(2,3
    92
    ----
    13,8
    13,8
    ----
    000
```

Here we divide 105 *units* by 46, and the quotient figure is 2 *units*.

We then subtract 46 times 2 units from the dividend, and 13 units remain. To this bring down the 8 tenths. This is divide

---

If the divisor be a mixed decimal, what is the process?

as if whole numbers, but the quotient 3 is 3 *tenths*, because the lowest order in the dividend is *tenths*. It is set in the quotient with the separatrix before it, and then 46 times ,3 (or 13,8) is taken from the dividend, and nothing remains.

Let the following sums be performed, and explained as above.

Divide	46,4	by	3,6	Divide	891,6	by	,2
"	,431	"	2,41	"	8,964	"	8,6
"	4,56	"	3,64	"	89,96	"	4,861
"	464,92	"	3,2649	"	8,641	"	,4169

The following then is the rule for Decimal Division.

RULE FOR DECIMAL DIVISION.

If the divisor is a whole number, divide as in common division, placing each quotient figure in the same order as the lowest order of the dividend taken.

If the divisor is a decimal, multiply by the denominator, and divide by the numerator, placing each quotient figure in the same order as the lowest order of the dividend taken.

If the divisor is a mixed decimal, change it to an improper decimal, and then proceed to multiply by the denominator and divide by the numerator.

N. B. The rule for multiplying and dividing Federal Money, is the same as for Decimals.

EXAMPLES.

How many times is $2,04 contained in $9,40?

Divide	$2,04	by	$,84
"	,02	"	8,41
"	2,41	"	19,24
"	324,07	"	64,81
"	20,46	"	,49

As it is found to be *invariably* the case that *the decimal orders in the divisor and quotient always equal those of the dividend*, the *common* rule for decimal division is formed on that principle, and may now be used.

COMMON RULE FOR DECIMAL DIVISION.

Divide as in whole numbers. Point off in the quotient enough decimals to make the decimal orders of the divisor and quotient

---

What is the rule for decimal division? What is the common rule for decimal division?

together equal to those of the dividend, counting every cipher annexed to the dividend, or to any remainder, as a decimal order of the dividend. If there are not enough figures in the quotient, prefix ciphers.

In pointing off by the above rule, let the teacher ask these questions.

How many decimals in the dividend? How many in the divisor? How many must be pointed off in the quotient, to make as many in the divisor and quotient, as there are in the dividend?

EXAMPLES.

At $,75 per bushel, how many bushels of oats can be bought for $14,23?

How much butter at 16 cents a pound, can be bought for $ 20?

A *half* cent can be written thus, $,005 (for 5 mills is half a cent, or 5 *thousandths* of a dollar.)

A *quarter* of a cent can be written thus, $,0025 (for ¼ of a cent is 25 *tenths of thousandths* of a dollar.)

At 12½ cents per hour, in how much time will a man earn $ 46.

At 6¼ cents per pint, how much molasses may be bought for $ 2?

At $,06 an ounce, how much camphor can be bought for $ 3?

At $,12½ a bushel, how much coal could be bought for $ 5?

Divide ,032 by ,005.

EXERCISES IN DECIMAL MULTIPLICATION AND DIVISION.

Multiply ,25 by ,003. Divide ,25 by ,003.

Multiply 3,4 by 2,68. Divide 3,4 by 2,68.

Multiply ,005 by ,005. Divide ,004 by 16,4.

If you buy 24 bushels of coal, at $ 09 per bushel, what does the whole cost?

If a man's wages be fifty hundredths of a dollar a day, what will it be a month?

What will be the cost of 25 thousandths of a cord of wood, at $ 2 a cord?

What will be the cost of twelve hundredths of a ton of hay, at $ 11 a ton?

If a man pays a tax of two mills on a dollar, how much must he pay if he is worth $ 350?

If a man pays $,06 a year for the use of each dollar he borrows of his neighbor, how much must he pay in a year if he borrows 264 dollars? How much in *two* years?

REDUCTION OF FRACTIONS TO WHOLE NUMBERS.

1. In ten fifths, how many units?
2. In fourteen sevenths, how many units?
3. Change fifteen fifths to units.
4. Change thirteen fourths to units, and what is the answer?
5. Change eighteen fourths to units, and what is the answer?
6. Change fourteen sixths to units.

It will be perceived, that in answering these questions, the pupil *divides the numerator by the denominator*. Thus in changing twelve fourths to units, the numerator twelve is divided by the denominator four. The above sums are to be performed mentally first, and the answers given, and then they are to be written, thus,

7. Change fourteen sixths to units.

Ans. $\frac{14}{6} = 14 \div 6 = 2\frac{2}{6}$.

Let the pupil be required to perform all the above sums, in this manner.

RULE FOR REDUCING FRACTIONS TO WHOLE NUMBERS.

Divide the numerator by the denominator; write the remainder, if there be any, over the denominator, and annex the fraction, thus formed, to the quotient.

EXAMPLES.

1. Reduce $\frac{39}{4}$ to a whole or mixed number Ans. $9\frac{3}{4}$.
2. Reduce $\frac{57}{6}$. Ans. $9\frac{3}{6}$. $\frac{85}{9}$. Ans. $9\frac{4}{9}$. $\frac{76}{5}$. Ans. $15\frac{1}{5}$. $\frac{19}{8}$. Ans. $2\frac{3}{8}$.
3. Reduce $\frac{158}{3}$. Ans. $52\frac{2}{3}$. $\frac{2825}{5}$. Ans. 565. $\frac{21825}{9}$. Ans. 2425.
4. Reduce $\frac{8121}{3}$. $\frac{6276}{6}$. $\frac{518432}{8}$. $\frac{915873}{7}$. $\frac{1325965}{5}$.
5. Reduce $\frac{987654321}{3}$. $\frac{700070007}{7}$. $\frac{600304002}{6}$.
6. Reduce $\frac{7112345499}{9}$. $\frac{4956350217}{7}$. $\frac{3322211136}{6}$. $\frac{59248321768}{8}$.

What is the rule for reducing fractions to whole numbers?

REDUCTION OF WHOLE NUMBERS TO FRACTIONS.

1. In three units, how many fourths, and how is the answer expressed in figures?
2. How many fifths in three units and two fifths, and how in the answer written?
3. Reduce 9 units to sixths.
4. Reduce 7 units and two twelfths to twelfths.

RULE FOR REDUCING WHOLE NUMBERS TO FRACTIONS.

Multiply the whole number by the denominator of the fraction to which it is to be reduced, and place the product over this denominator. If there is with the units, a fraction of the same denominator, add the numerator of this fraction to the product, before placing it over the denominator.

EXAMPLES.

1. How many 4ths in 1? How many in $1\frac{1}{4}$? In $1\frac{2}{4}$? In $1\frac{3}{4}$?
2. How many 5ths in 1? In 5? In $1\frac{1}{5}$? In $1\frac{3}{5}$? In $7\frac{4}{5}$?
3. How many 7ths in 7? In 8? In 12? In $7\frac{3}{7}$? In $5\frac{6}{7}$?
4. How many 12ths in $9\frac{5}{12}$? In $7\frac{6}{12}$? In $3\frac{4}{12}$? In $5\frac{5}{12}$? In $8\frac{7}{12}$?
5. How many 6ths in 3? In 4? In $5\frac{2}{6}$? In $7\frac{5}{6}$? In 8? In $9\frac{4}{6}$? In 12?
6. How many 27ths in 3? In 2? In $5\frac{9}{27}$? Ans. $\frac{81}{27}$. $\frac{54}{27}$. $\frac{144}{27}$.
7. How many 19ths in 15? In $13\frac{3}{19}$? In $17\frac{18}{19}$? Ans. $\frac{285}{19}$. $\frac{250}{19}$. $\frac{341}{19}$.

REDUCTION OF VULGAR TO DECIMAL FRACTIONS.

Decimal Fractions are generally used in preference to Vulgar, because it is so easy to multiply and divide by their denominators.

What is the rule for reducing whole numbers to fractions?

Vulgar Fractions can be changed to Decimals by a process which will now be explained.

In this process, the numerator is to be considered as *units* divided by the denominator.

Thus $\frac{3}{4}$ is 3 *units* divided by 4, for $\frac{3}{4}$ is a fourth of 3 units.

We can change these 3 units to an improper decimal thus, 3'0 (30 tenths), and then divide by 4; remembering that the *quotient is of the same order as the dividend.*

```
4)3'0(,75
  2'8
  ---
   ,20
   ,20
```

Thus the 30 *tenths* are divided by 4, and the answer is 7 *tenths*, which is placed in the quotient, with a separatrix prefixed. 4 times 7 tenths (or 28 tenths) are then subtracted, and the remainder is ,2. This, in order to divide it by 4, must have a cipher annexed, making it 20 *hundredths.* The quotient of this is 5 *hundredths*, and no remainder.

(In performing this process, particular care must be taken in using the separatrix, both for proper and improper decimals.)

Let $\frac{2}{8}$ be reduced in the same way.

The two units are first changed to an improper decimal, thus:

```
8)2'0(,25
  1'6
  ---
   ,40
   ,40
   ---
    00
```

We proceed thus. 20 tenths divided by 8, is 2 *tenths*, which is placed in the quotient. 8 times ,2 or 16 tenths (1'6) is then subtracted, and ,4 remain.

This is changed to 40 hundredths (,40) by adding a cipher, and then divided by 8. The quotient is 5 *hundredths*, which is put in the quotient and there is no remainder.

NOTE. After 3 or 4 figures are put in the quotient, if there still continues to be a remainder, it is not needful to continue the division, but merely to put the *sign of addition* in the *quotient* to show that more figures might be added.

EXAMPLES.

Reduce $\frac{2}{12}$ to a decimal, and explain as above.

Reduce $\frac{4}{6}$ $\frac{2}{5}$ $\frac{4}{8}$ $\frac{2}{11}$ $\frac{8}{13}$ $\frac{4}{7}$ $\frac{8}{9}$ $\frac{6}{8}$ each to a decimal of the same value.

Let the pupil be required to explain sums of this kind as directed above, until perfectly familiar with the principle.

When fractions of *dollars and cents* are expressed, their decimal value is found by the same process.

For example, change $\frac{1}{2}$ a dollar to a decimal.

Here the 1 of the numerator, is *one dollar*, divided by 2. By adding a cipher to this 1 and using the inverted separatrix, the dollar is changed to 10 dimes, and when this is divided by 2, the answer is 5; which being of the same order as the dividend, is 5 *dimes*.

The answer is to be written with the sign of the dollar before it, thus $ 0,5.

The only difference between the answer when ½ is reduced to a decimal, and when ½ *a dollar* is reduced to a decimal, is simply the use of the sign of a dollar ($) and a cipher in the dollar order.

1. Reduce ½ to a decimal. *Ans.* ,5.
2. Reduce ½ a dollar to a decimal. *Ans.* $ 0,5.
3. Change ⅛ of a dollar to a decimal. *Ans.* $ 0,125.
4. Change $\frac{1}{16}$ of a dollar to a decimal. *Ans.* $ 0,0625.

In this last sum there must be *two* ciphers added to the numerator, changing the 1 dollar to *cents*, instead of *dimes*; and in this case a cipher is put in the order of dimes, and the quotient (being of the same order as the dividend) is placed in the order of *cents*.

5. Reduce ⅕ of a dollar to a decimal. *Ans.* $ 0,2.
6. Reduce ⅝ of a dollar to a decimal. *Ans.* $ 0,625.
7. Reduce $\frac{3}{16}$ of a dollar to a decimal. *Ans.* $ 0,1871.
8. Reduce $\frac{1}{20}$ to the decimal of a dollar. *Ans.* $ 0,05.

RULE FOR THE REDUCTION OF VULGAR TO DECIMAL FRACTIONS.

Change the numerator to an improper decimal, by annexing ciphers and using an inverted separatrix. Divide by the denominator, placing each quotient figure in the same order as the lowest order of the part divided.

1. Reduce $\frac{1}{625}$ to a decimal. *Ans.* .0016.
2. Reduce $\frac{7}{250}$ to a decimal. *Ans.* .028.
3. Reduce $\frac{9}{160}$ to a decimal. *Ans.* .05625.
4. Reduce ⅓ to a decimal. *Ans.* .3333333 +

NOTE. We see here, that we may go on for ever, and the decimal will continue to repeat 33, &c. therefore, the sign of addition + in such cases may be added, as soon as it is found that the same number continues to recur in the quotient.

Why are decimal fractions used in preference to vulgar? What is the rule for reducing vulgar to decimal fractions?

REDUCTION OF FRACTIONS TO A COMMON DENOMINATOR.

Before explaining this process, it must be remembered that $\frac{3}{3}$ $\frac{4}{4}$ $\frac{5}{5}$ $\frac{6}{6}$ &c. or a fraction which has the numerator and denominator alike, is the same as a *unit*. If therefore we take a *fourth* of $\frac{4}{4}$ it is the same as taking a fourth of *one*. If we take a *sixth* of $\frac{6}{6}$ it is the same as taking a sixth of *one*.

If we take $\frac{2}{3}$ of $\frac{6}{6}$ it is the same as taking $\frac{2}{3}$ of *one*.

Whenever therefore we wish to change one fraction to another, without altering its value, we suppose a *unit* to be changed to a *fractional form*, and then take such a part of it, as is expressed by the fraction to be changed,

For example, if we wish to change $\frac{1}{4}$ to *twelfths*, we *change a unit* to twelfths and then take $\frac{1}{4}$ of it, and we have $\frac{1}{4}$ of $\frac{12}{12}$, which is the same as $\frac{1}{4}$ of *one*.

If we wish to change $\frac{1}{2}$ to *eighths*, we change a unit to $\frac{8}{8}$ and then take $\frac{1}{2}$ of it, for $\frac{1}{2}$ of $\frac{8}{8}$ is the same as $\frac{1}{2}$ of *one*.

Change $\frac{2}{3}$ to twelfths, thus, a unit is $\frac{12}{12}$. *One* third of $\frac{12}{12}$ is $\frac{4}{12}$. *Two* thirds is twice as much, or $\frac{8}{12}$. Then $\frac{2}{3}$ are $\frac{8}{12}$.

Change $\frac{4}{5}$ to *twentieths*. A unit is $\frac{20}{20}$. *One* fifth of $\frac{20}{20}$ is $\frac{4}{20}$. *Four* fifths is four times as much, or $\frac{16}{20}$.

Change the following fractions, and state the process in the same way.

Change $\frac{2}{8}$ to twenty-fourths.
Change $\frac{5}{6}$ to twelfths.
Reduce $\frac{3}{9}$ to twenty-sevenths.
Reduce $\frac{6}{8}$ to sixty-fourths.
Reduce $\frac{4}{5}$ to twenty-fifths.
Reduce $\frac{8}{9}$ to twenty-sevenths.
Reduce $\frac{7}{9}$ to thirty-sixths.
Reduce $\frac{5}{7}$ to forty-ninths.
Reduce $\frac{3}{12}$ to thirty-sixths.
Rèduce $\frac{4}{8}$ to sixteenths.
Reduce $\frac{2}{10}$ to fortieths.

Reduce $\frac{10}{11}$ to thirty-thirds.
Reduce $\frac{4}{6}$ to thirty-sixths.
Reduce $\frac{2}{3}$ and $\frac{3}{4}$ each to twelfths.
Reduce $\frac{3}{5}$ and $\frac{4}{10}$ each to twentieths.
Reduce $\frac{1}{2}$ $\frac{2}{6}$ and $\frac{3}{4}$ each to twelfths.
Reduce $\frac{1}{5}$ $\frac{2}{10}$ and $\frac{5}{20}$ each to fortieths.
Reduce $\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$ and $\frac{8}{32}$ each to sixty-fourths.
Reduce $\frac{2}{3}$ $\frac{4}{6}$ $\frac{8}{12}$ and $\frac{12}{24}$ each to forty-eighths.

In the above examples it is seen that *when several fractions are to be reduced to a common denominator*, a *unit* is changed first to a fractional form with the required denominator. Then it is *divided by the denominator* of each fraction, to obtain *one part*, and multiplied by the numerator, to obtain the *required number of parts.*

Thus changing $\frac{3}{4}$ and $\frac{2}{6}$ each to twelfths, we first change a unit to a fraction with the required denominator 12; thus, $\frac{12}{12}$. We then divide it by the denominator of $\frac{3}{4}$, to obtain *one* fourth, and multiply the answer by 3, to obtain *three* fourths. In like manner with the $\frac{2}{6}$. We divide $\frac{12}{12}$ by the denominator 6, to obtain *one* sixth, and multiply by the numerator to obtain *two* sixths.

In changing fractions to common denominators then, the *unit* must be changed to that fractional form which will enable us to divide it by *all* the denominators of the fractions (which are to be reduced) *without remainder.*

Thus if we wish to reduce $\frac{2}{7}$ and $\frac{3}{5}$ to a common denominator, we cannot reduce them to *twelfths*, because $\frac{12}{12}$ cannot be divided by either the denominator 5, or 7, without remainder. We must therefore seek a number that can be thus divided, both by 7 and 5. 35 is such a number. We now take $\frac{3}{5}$ of $\frac{35}{35}$ and $\frac{2}{7}$ of $\frac{35}{35}$ and the two fractions are then reduced to a common denominator.

ONE MODE OF REDUCING FRACTIONS TO A COMMON DENOMINATOR.

Change a unit to a fraction whose denominator can be divided by all the denominators of the fractions to be reduced, without remainder. Divide this fraction by the denominator of each fraction to obtain one part, and multiply by the numerator to obtain the required number of parts.

FURTHER EXAMPLES FOR MENTAL EXERCISE.

Reduce $\frac{2}{5}$ $\frac{3}{6}$ and $\frac{2}{3}$ to a common denominator. Let the unit be reduced to $\frac{30}{30}$.

Reduce $\frac{4}{6}$ $\frac{1}{2}$ $\frac{2}{3}$ to a common denominator. Let the unit be reduced to $\frac{12}{12}$.

Reduce $\frac{5}{8}$ $\frac{3}{4}$ $\frac{1}{2}$ and $\frac{4}{16}$ to a common denominator.

Reduce $\frac{2}{3}$ $\frac{4}{6}$ $\frac{5}{9}$ to a common denominator.

Reduce $\frac{7}{9}$ $\frac{5}{6}$ $\frac{3}{4}$ to a common denominator.

Reduce $\frac{9}{12}$ $\frac{4}{6}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{2}$ to a common denominator.

But there is another method of reducing fractions to a common denominator, which is more convenient for operations on the slate. When a fraction has both its terms (that is, its *numerator* and *denominator*) multiplied by the same number, its *value* remains the *same.*

For example; multiply both the numerator and denominator of $\frac{2}{3}$ by 4, and it becomes $\frac{8}{12}$. But $\frac{2}{3}$ and $\frac{8}{12}$ are the *same value*, with *different names.*

The effect, then, of multiplying both terms of a fraction by the same number, is to *change their name, but not their value.*

If therefore we have two fractions, and wish to change them so as to have both their denominators alike, we can do it by multiplication.

For example;

Let $\frac{2}{3}$ and $\frac{4}{9}$ be changed, so as to have the same denominator. This can be done by multiplying both terms of the $\frac{2}{3}$ by 9, and of $\frac{4}{9}$ by 3. The answers are $\frac{18}{27}$ and $\frac{12}{27}$, and the value of both fractions is unaltered.

In this case both terms of each fraction were multiplied *by the denominator of the other fraction.*

Let the following fractions be reduced to a common denominator in the same way.

1. Reduce $\frac{3}{5}$ and $\frac{2}{7}$ to a common denominator. Multiply the $\frac{3}{5}$ by the denominator 7, and the $\frac{2}{7}$ by the denominator 5.

2. Reduce $\frac{3}{9}$ and $\frac{4}{6}$ to a common denominator.

What is a rule for reducing fractions to a common denominator? What is the effect of multiplying both terms of a fraction by the same number?

3. Reduce $\frac{2}{8}$ and $\frac{5}{6}$ to a common denominator.

4. Reduce $\frac{2}{10}$ and $\frac{5}{7}$ to a common denominator.

The same course can be pursued, where there are *several* fractions, to be reduced to a common denominator.

Thus if $\frac{1}{2}$ $\frac{2}{3}$ and $\frac{3}{4}$ are to be reduced to a common denominator, we can multiply both terms of the $\frac{1}{2}$ first by the denominator 3, and then multiply both terms of the answer by the denominator 4, and it becomes $\frac{12}{24}$, and its value remains unaltered. For $\frac{1}{2}$ and $\frac{12}{24}$ have the same value with a different name.

Then we can multiply both terms of the $\frac{2}{3}$, first by the denominator 2, and then by the denominator 4, and it becomes $\frac{16}{24}$, and its value remains unaltered.

Then $\frac{3}{4}$ may be multiplied, first by the denominator 2, and then by the denominator 3, and it becomes $\frac{18}{24}$, and its value is unaltered.

The three fractions $\frac{1}{2}$ $\frac{2}{3}$ and $\frac{3}{4}$ are thus changed to $\frac{12}{24}$ $\frac{16}{24}$ and $\frac{18}{24}$, which have a common denominator, and yet their value is unaltered.

But instead of multiplying each fraction, by *each separate* denominator, it is a shorter way to multiply by *the product of these denominators.*

Thus in the above example, instead of multiplying the $\frac{1}{2}$, first by 3, and then the answer by 4, it is shorter to multiply by 12 (the product of 3 and 4), and the answer will be the same.

In like manner, if we were to reduce $\frac{3}{4}$ $\frac{3}{6}$ and $\frac{4}{7}$ to a common denominator, we should multiply both terms of each fraction by the denominators of all the other fractions. But instead of each denominator *separately*, as multiplier, we can take the *product* of them for the multiplier.

Reduce $\frac{2}{5}$ $\frac{3}{4}$ and $\frac{1}{3}$ to a common denominator.

Here both terms of the $\frac{2}{5}$ are first multiplied by the *product* of the other two denominators (which is 12). Then both terms of $\frac{3}{4}$ are multiplied in the same way by the product of the other two denominators (15). Then both terms of $\frac{1}{3}$ are multiplied by the product of the other two denominators (20).

RULE FOR REDUCING FRACTIONS TO A COMMON DENOMINATOR.

Multiply both terms of each fraction by the product of all the denominators except its own.

Reduce $\frac{1}{3}$ $\frac{2}{4}$ $\frac{3}{8}$ to a common denominator.

Reduce $\frac{7}{8}$ $\frac{9}{10}$ and $\frac{11}{12}$ to a common denominator.

Ans. $\frac{840}{960}$ $\frac{864}{960}$ and $\frac{880}{960}$.

Reduee $\frac{1}{2}$ $\frac{2}{3}$ $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{144}{288}$ $\frac{192}{288}$ $\frac{240}{288}$ and $\frac{252}{288}$.

Reduce $\frac{4}{5}$ $\frac{6}{24}$ and $\frac{4}{10}$ to a common denominator.

Reduce $\frac{3}{4}$ $\frac{5}{6}$ and $12\frac{1}{3}$ to a common denominator.

Ans. $\frac{54}{72}$ $\frac{60}{72}$ $\frac{888}{72}$.

Reduce $\frac{2}{9}$ $\frac{3}{4}$ and $\frac{5}{8}$ of $\frac{11}{12}$ to a common denominator.

Ans. $\frac{768}{3456}$ $\frac{2592}{3456}$ $\frac{1980}{3456}$.

REDUCTION OF FRACTIONS TO THEIR LOWEST TERMS.

What is the difference between $\frac{1}{2}$ and $\frac{2}{4}$?

Ans. They express the *same value*, by *different names*.

Which fraction has the *smallest numbers* employed to express its value?

In the two fractions $\frac{3}{5}$ and $\frac{6}{10}$ is there any difference in the value?

Which fraction has its value expressed by the smallest numbers?

A fraction is reduced to its lowest terms, when its value is expressed by the smallest numbers which can be used, to express that value.

For example, $\frac{3}{4}$ is reduced to its lowest terms, because no smaller numbers than 3 and 4 can express this value.

The value of a fraction is not altered if both terms of it are divided by the *same number*.

Thus if $\frac{4}{8}$ has both its terms divided by 2, it becomes $\frac{2}{4}$ and the value remains the same. If it is divided by 4, it becomes $\frac{1}{2}$ and its value remains unaltered.

When it was divided by 2, it was not reduced to its lowest

What is the rule for reducing fractions to a common denominator? When is a fraction reduced to its lowest terms?

terms, because smaller numbers can express the same value, as $\frac{2}{4}$. But when it was divided by 4, it was reduced to its lowest terms, because no smaller numbers than 1 and 2 can express its value.

The *shortest* way to reduce a fraction to its lowest terms is, to divide it by the *largest number* which will divide both terms, *without a remainder*.

Any number which will divide two or more numbers without a remainder is called a *common measure*, and the largest number which will do this, is called the *greatest common measure*.

In many operations it saves much time to have a fraction reduced to its lowest terms. Thus for example, if we are to multiply 3429 by $\frac{27}{36}$ it would be much easier to reduce the fraction to $\frac{3}{4}$ (which are its lowest terms) and then multiply.

There are many fractions which can be reduced to their low est terms without much trouble. For example let the pupil reduce these fractions.

Reduce $\frac{2}{4}$ $\frac{3}{6}$ $\frac{5}{15}$ $\frac{3}{18}$ $\frac{6}{24}$ to their lowest terms.

But there are many fractions, which it is much more difficult to reduce. Thus if we wish to reduce $\frac{234}{1836}$ to its lowest terms, we could not so readily do it.

In such a case as this there are *two* ways of doing it; the first is as follows.

RULE FOR REDUCING A FRACTION TO ITS LOWEST TERMS.

Divide the terms of the fraction by any number that will divide both, without a remainder. Divide the answer obtained in the same way. Continue thus, till no number can be found that will divide both terms without a remainder.

Thus, Reduce $\frac{234}{1836}$ to its lowest terms.

N. B. The brackets at the right of the fractions show that *both terms* of the fraction are to be divided by the divisor, and not the *fraction itself*, as in the division of fractions.

$$\left.\frac{234}{1836}\right) \div 3 = \frac{78}{612}$$
$$\left.\frac{78}{612}\right) \div 2 = \frac{39}{306}$$
$$\left.\frac{39}{306}\right) \div 3 = \frac{13}{102} \quad \textit{Answer.}$$

What is the shortest way to reduce a fraction to its lowest terms? What is meant by a common measure? What is the rule for reducing a fraction to its lowest terms?

In the above process, both terms of the fraction $\frac{234}{1836}$ are divided by 3; the answer is divided by 2; and this answer again is divided by 3.

The last answer is $\frac{13}{102}$ which cannot have both terms divided by any number without a remainder.

The other method of reducing a fraction to its lowest terms, is first to find the number which is the *greatest common measure*, and then to divide the fraction by this number.

The following is the method of finding the greatest common measure, and reducing to the lowest terms.

Reduce $\frac{21}{35}$ to its lowest terms.

The denominator is first placed as a dividend, and the numerator, as a divisor; (below.) After subtracting, the *remainder* (14) is used for the *divisor*, and the *first divisor* (21) is used for the *dividend*. This process of dividing the *last divisor* by the *last remainder* is continued till nothing remains. The *last divisor* (7) is the *greatest common measure*.

We then take the fraction $\frac{21}{35}$ and divide both terms by 7, the greatest common measure, and it is reduced to its lowest terms, viz. $\frac{3}{5}$.

```
21)35(1
   21
   --
   14)21(1
      14
      --
       7)14(2
         14
         --
         00
```

$\frac{21}{35}) \div 7 = \frac{3}{5}$.

RULE FOR FINDING THE GREATEST COMMON MEASURE OF A FRACTION AND REDUCING IT TO ITS LOWEST TERMS.

Divide the greater number by the less. Divide the divisor by the remainder, and continue to divide the last divisor by the last remainder, till nothing remains. The last divisor is the greatest common measure, by which both terms of the fraction are to be divided, and it is reduced to its lowest terms.

Reduce the following Fractions to their lowest terms.

What is the rule for finding the greatest common measure of a fraction?

$\frac{486}{9720}$; $\frac{144}{1728}$; $\frac{324}{648}$; $\frac{1429}{2858}$; $\frac{1644}{2192}$; $\frac{468}{1184}$; $\frac{4746}{38433}$; $\frac{806}{42315}$; $\frac{1158}{2702}$; $\frac{3578164}{32203476}$.

Ans. $\frac{1}{20}$; $\frac{1}{12}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{117}{296}$; $\frac{1582}{12811}$; $\frac{2}{105}$; $\frac{3}{7}$; $\frac{1}{9}$.

Reduce the following; $\frac{386}{948}$; $\frac{4932}{8764}$; $\frac{12345}{678910}$; $\frac{24687}{342854}$; $\frac{83808}{95636}$; $\frac{39972}{812322}$; $\frac{998811}{99988811}$; $\frac{103284}{7328472}$.

REDUCTION OF FRACTIONS FROM ONE ORDER TO ANOTHER ORDER.

It will be recollected that in changing *whole numbers* from one order to another, it was done by multiplication and division.

Thus, if 40 shillings were to be changed to pounds, we *divided* them by the number of shillings in a pound, and if £2 were to be reduced to shillings, we *multiplied* them by the number of shillings in a pound.

The same process is used in changing *fractions* of one order to fractions of another order.

Thus, if we wish to change $\frac{1}{200}$ of a £ to a fraction of the *shilling* order, we multiply it by 20, making it $\frac{20}{200}$. For $\frac{20}{200}$ of a shilling is the same as $\frac{1}{200}$ of a pound.

If we wish to change $\frac{40}{200}$ of a *shilling*, to the same value in a fraction of the *pound* order, we *divide* $\frac{40}{200}$ by 20, making it $\frac{2}{200}$. (This could also be *divided* by *multiplying its denominator* by 20.)

If then we wish to change a fraction of a *lower* order to the same value in a *higher* order, we must *divide* the fraction, by *multiplying the denominator*, by that *number of units* (of the order to which the fraction belongs) which make a unit of the order to which it is to be changed.

Thus if we wish to change $\frac{2}{8}$ of a penny to the same value in the fraction of a shilling, we multiply its denominator by 12, making it $\frac{2}{96}$ of a shilling. If we wish to change this to the same value in a fraction of the pound order, we must now multiply its denominator by the number of shillings which make a pound, making it $\frac{2}{1920}$ of a pound. It must be remembered that multiplying the *denominator* of a fraction, is *dividing* the fraction.

If, on the contrary, we wish to change a fraction of a *higher* order to one of the same value in a *lower* order, we must multiply.

Thus, to change $\frac{2}{134}$ of a shilling to the penny order, we must multiply it by 12. This we do *by multiplying its numerator* by 12, and the answer is $\frac{24}{134}$. For as there are 12 times as many *whole* pence in a whole shilling, so there are 12 times as many $\frac{2}{134}$ of a penny in $\frac{2}{134}$ of a shilling.

RULE FOR REDUCING FRACTIONS OF ONE ORDER TO ANOTHER ORDER.

To reduce a fraction of a higher order to one of a lower order.

Multiply the fraction by that number of units of the next lower order, which are required to make one unit of the order to which the fraction belongs. Continue this process till the fraction is reduced to the order required.

To reduce a fraction of a lower to one of a higher order.

Divide the fraction (by multiplying the denominator) by the number of units which are required to make one unit of the next higher order. Continue this process till the fraction is reduced to the order required.

EXAMPLES.

Reduce $\frac{7}{1387}$ of a guinea, (or of 28 shillings,) to the fraction of a penny.

Reduce $\frac{4}{5}$ of a guinea to the fraction of a pound.

Reduce $\frac{3}{38}$ of a pound Troy, to the fraction of an ounce.

Reduce $\frac{3}{10}$ of an ounce to the fraction of a pound Troy.

Reduce $\frac{2}{25}$ of a pound Avoirdupoise to the fraction of an ounce.

A man has $\frac{2}{648}$ of a hogshead of wine, what part of a pint is it?

A vine grew $\frac{2}{7858}$ of a mile, what part of a foot was it?

Reduce $\frac{3}{8}$ of $\frac{2}{3}$ of a pound to the fraction of a shilling.

Reduce $\frac{2}{3}$ of $\frac{2}{5}$ of 3 shillings, to the fraction of a pound.

REDUCTION OF FRACTIONS OF ONE ORDER, TO UNITS OF A LOWER ORDER.

It is often necessary to change a fraction of one order, to units of a lower order. For example, we may wish to change $\frac{3}{4}$ of a unit of the *pound* order, to units of the *shilling* order.

...a rule for reducing fractions of one order to another order?

This $\frac{2}{3}$ of a £ is 2 *pounds divided by* 3. These 2 pounds are changed to *shillings*, by multiplying by 20, and then divided by 3; and the answer is $13\frac{1}{3}$ shillings. This $\frac{1}{3}$ of a shilling may be reduced to pence in the same way, for $\frac{1}{3}$ of a shilling is 1 shilling divided by 3. This 1 shilling can be changed to *pence*, and then divided by 3, the answer is 4 *pence*.

RULE FOR FINDING THE VALUE OF A FRACTION IN UNITS OF A LOWER ORDER.

Consider the numerator as so many units of the order in which it stands, and then change it to units of the order in which you wish to find the value of the fraction. Divide by the denominator, and the quotient is the answer, and is of the same order as the dividend.

EXAMPLES.

1. How many ounces in $\frac{3}{5}$ of a lb. Avoirdupoise?
2. How many days, hours and minutes, in $\frac{4}{5}$ of a month?
3. What is the value of $\frac{5}{8}$ of a yard?
4. What is the value of $\frac{3}{12}$ of a ton?
5. How many pence in $\frac{2}{3}$ of a lb.?
6. How many drams in $\frac{3}{5}$ of a lb. Avoirdupoise?
7. How many grains in $\frac{2}{3}$ of a lb. Troy weight?
8. How many scruples in $\frac{3}{4}$ of a lb. Apothecaries weight?
9. How many pints in $\frac{2}{7}$ of a bushel?

REDUCTION OF UNITS OF ONE ORDER TO FRACTIONS OF ANOTHER ORDER.

It is necessary often to reverse the preceding process, and change units to fractions of another order. For example, to change 13s. 4d. to a fraction of the *pound* order.

To do this we change the 13s. 4d. to units of the lowest order mentioned, viz. 160 *pence*. This is to be the *numerator* of the fraction. We then change a unit of the pound order to pence (240) and this is the *denominator* of the fraction. The answer is $\frac{160}{240}$ of a pound.

What is the rule for finding the value of a fraction in units of a lower order?

For if 13s. 4d. is 160 pence, and a £ is 240 pence, then 13s. 4d. is $\frac{160}{240}$ of a pound.

RULE FOR REDUCING UNITS OF ONE ORDER TO FRACTIONS OF ANOTHER ORDER.

Change the given sum to units of the lowest order mentioned, and make them the numerator.

Change a unit of the order to which the sum is to be reduced, to units of the same order as the numerator, and place it for the denominator.

EXAMPLES.

Reduce 6 oz. 4 pwt. to the fraction of a pound Troy.
Reduce 3 days, 6 hours, 9 minutes to the fraction of a month.
Reduce 2 cwt. 2 qrs. 16 lbs. to the fraction of a ton.
Reduce 2 lb. 4 oz. to the fraction of a cwt.

REDUCTION OF A COMPOUND NUMBER TO A DECIMAL FRACTION.

It is often convenient to change a *compound number* to a *decimal fraction*.

Thus we can reduce 1 oz. 10 pwt. to a *decimal of the pound order*.

Let the figures be placed thus, and the process will be explained below. The 10 pwts. are first written, and then the 1 oz. set under.

```
20)10‘0 pwt.
12) 1‘5 oz.
    ‘125 lb.
```

We first change the lowest order (10 pwts.) to an *improper* decimal, thus, 10‘0. Now as 20 pwts. make an oz. there are but *one twentieth* as many *ounces* in a sum as there are *pennyweights*.

For the same reason, in any sum there are but one twentieth as many *tenths of an ounce* as there are *tenths of a pennyweight*.

As there are then 100 *tenths of a pwt.* in this sum, if we take one twentieth of them, we shall find how many *tenths of an oz.* there are.

We therefore divide the 10‘0 pwts. by 20, and the amount is

What is the rule for reducing units of one order to fractions of another order?

,5. This ,5 is placed (beside the 1 oz. of the sum) under the 10'0 pwts., and thus, instead of reading the sum as 1 oz. 10 pwts., we read it as 1,5 oz., or 1 oz. and five tenths of an oz.

As the pwts. are thus reduced to the decimal of an oz. we now reduce the 1,5 oz. to the decimal of a lb. in the same way.

We make the 1,5 an improper decimal, thus, 1'5 (15 *tenths*) of an oz.

Now as there are 12 oz. in a lb. there are but *one twelfth* as many *tenths of a lb.* in a sum, as there are *tenths of an oz.* We therefore divide the 15 tenths of an oz. by 12, and the answer is ,1 of a lb. and 3 left over. This 3 is reduced to hundredths by adding a cipher and dividing it again. The quotient is 2 *hundredths.* The next remainder is changed to thousandths in the same way, and the answer is ,125 of a lb.

RULE FOR CHANGING A COMPOUND NUMBER TO A DECIMAL.

Change the lowest order to an improper decimal. Divide it by the number of units of this order, which are required to make a unit of the next higher order, and set the answer beside the units of the next higher order. Repeat this process till the sum is brought to the order required.

EXAMPLES.

Reduce 10s. 4d. to the decimal of a £.
Reduce 8s. 6d. 3 qrs. to the decimal of a £.
Reduce 17 hrs. 16 min. to the decimal of a day.
Reduce 3 qrs. 2 nā. to the decimal of a yd.
Reduce 32 gals. 4 qts. to the decimal of a hogshead.
Reduce 10d. 3 qrs. to the decimal of a shilling.

REDUCTION OF A DECIMAL TO UNITS OF COMPOUND ORDERS.

The preceding process can be reversed, and a decimal of one order, be changed back to units of other orders.

Thus, if we have ,125 of a lb. Troy, we can change it to units of the oz. and pwt. order.

What is the rule for changing a compound number to a decimal?

In performing the process, we place the figures thus

We reason thus. In ,125 of a lb. there must be 12 times as many thousandths of an oz. (for 12 oz. = 1 lb.) We therefore multiply by 12, and point off according to rule, and the answer is 1 oz. and 500 thousandths of an oz.

```
  ,125 lb.
    12
 ------
 1,500 oz.
    20
 ------
10,000 pwt.
```

Now as we have found how many oz. there are, we must find how many pwts. there are in the ,500 of an oz. There must be 20 times as many thousandths of a pwt. as there are thousandths of an oz. therefore multiply the *decimal only*, by 20, and point off according to rule, and we find there are 10 pwts.

We have thus found that in ,125 of a lb. there are 1 oz. and 10 pwts.

RULE FOR CHANGING A DECIMAL OF ONE COMPOUND ORDER, TO UNITS OF OTHER ORDERS.

Multiply the decimal by the number of units of the next lower order which are required to make one unit of the order in which the decimal stands.

Point off according to rule, and multiply the decimal part of the answer in the same way, pointing off as before. Thus till the sum is brought into the order required. The units of each answer make the final answer.

In ,1257 of a £ how many shillings, pence and farthings?
What is the value of ,2325 of a ton?
What is the value of ,375 of a yard?
What is the value of ,713 of a day?
What is the value of ,15334821 of a ton?

REDUCTION OF CURRENCIES.

There are few exercises in Reduction, of more *practical* use than the *Reduction of Currencies*, by which a sum in one currency is changed to express the same value in another currency.

An example of this kind of reduction occurs, when the value of $1 is expressed in British currency thus, 4s. 6d.

What is the rule for changing a decimal of one compound order to units of other orders? What is reduction of currencies?

The necessity for using this process in this country, results from the following facts.

Before the independence of the U. States, business was transacted in the currency of Great Britain. But at various times, the governments of the different States put bills into circulation, which constantly lessened in value, until they became very much depreciated. For example, a bill which was called *a pound* or twenty shillings, British currency, was reduced to be worth only *fifteen* shillings, in the New England states.

This depreciation was greater in some states than it was in others, and the result is, that pounds, shillings and pence have different values in different states.

12 pence make a shilling, and 20 shillings make a pound in all cases, but the *value* of a penny, a shilling, or a pound, depends upon the *currency* to which it belongs.

The following table shows the relative value of the several currencies, by showing the value of *one dollar* in each of the different currencies.

VALUE OF ONE DOLLAR IN EACH OF THE DIFFERENT CURRENCIES.

$1	equals	6*s*.	New England currency.
$1	"	8*s*.	New York currency.
$1	"	7*s*. 6*d*.	Pennsylvania currency.
$1	"	4*s*. 8*d*.	Georgia currency.
$1	"	4*s*. 6*d*.	Sterling money, or English currency.
$1	"	5*s*.	Canada currency.
$1	"	4*s*. 10½*d*.	Irish currency.
$1	"	£2. 14*s*.	Scotch currency.

VALUE OF ONE POUND OF EACH OF THE DIFFERENT CURRENCIES, EXPRESSED IN FEDERAL MONEY.

£1 N. England currency	equals	$3,333⅓
£1 N. York currency	"	$2,50
£1 Pennsylvania currency	"	$2,666⅔
£1 Georgia currency	"	$4,285 5/7
£1 Sterling money	"	$4,444 4/9
£1 Canada currency	"	$4,00
£1 Irish currency	"	$4,10⅔
£1 Scotch currency	"	$0,370 10/27

What is the cause of the difference in the currencies of the several states?

The following sums for mental exercise, will be found of much *practical* use, and should be practised till they can be *readily* answered.

EXAMPLES IN NEW ENGLAND CURRENCY, FOR MENTAL EXERCISE.

1. If 6 shillings equal a dollar or 100 cents, how many cents in 3 shillings? in 2 shillings? in 1 shilling? in 4 shillings? in 5 shillings?

2. If 1 shilling is $16\frac{2}{3}$ cts. how many cents in 6 *pence*? in 3 pence? in 9 pence? in 4 pence? in 7 pence? in 8 pence? in 11 pence?

3. How many cents in 1s. 6d.? in 1s. 9d.? in 1s. 3d.? in 2s. 6d.? in 2s. 9d.? in 3s. 4d.? in 5s. 6d.? in 7s. 6d.? in 8s. 6d.? in 9s.? in 9s. 6d.? in 10s. 6d.? in 11s.? in 11s. 6d.? in 12s.?

4. If 6d. is $8\frac{1}{3}$ cts. how many cents is 3d.? how many is 1d.? how many is 2d.?

5. If you buy 8 yds. of ribbon at 1s. 6d. per yard, how many cents will the whole cost?

6. If you buy $2\frac{1}{4}$ yds. of muslin, at 2s. 6d. per yd. how much will it cost in dollars and cents?

7. If you buy $3\frac{1}{2}$ yds. of ribbon at 1s. 9d. per yd. how much will it cost in dollars and cents?

8. If you buy a brush for 2s. 3d. and a penknife for 4s. 6d. and a comb for 1s. 6d. how much is given for the whole in dollars and cents?

9. If you pay 3s. 6d. for scissors, 2s. 4d. for a thimble, and 1s. 9d. for needles, how much will the whole cost?

10. If linen is 4s. 6d. per yard, how much will $4\frac{3}{4}$ yds. cost?

11. If a piece of calico is 2s. 3d. per yd. how much will $6\frac{1}{4}$ yds. cost?

12. If muslin is 4s. 6d. per yd. what will $2\frac{3}{4}$ yds. cost?

13. How much is $11\frac{1}{2}$d.? $10\frac{1}{2}$d.? $9\frac{1}{2}$d.? $8\frac{1}{2}$d.? $7\frac{1}{2}$d.? $12\frac{1}{2}$d.? $16\frac{1}{2}$d.?

EXAMPLES IN NEW YORK CURRENCY, FOR MENTAL EXERCISE.

1. If a dollar in New York currency is 8s. how many cents in 4s.? in 2s.? in 1s.? in 5s.? in 6s.? in 7s.? in 9s.? in 10s.? in 11s.? in 12s.? in 13s.? in 14s.? in 15s.? in 16s.?

2. If one shilling is $12\frac{1}{2}$ cts. how many cents in 6d.? in 3d.? in 1d.? in 2d.? in 4d.? in 7d.? in 8d.? in 9d.? in 10d.? in ?

3. How many cents is 1s. 6d. N. York currency? is 2s. 6d.? is 3s. 6d.? is 5s. 3d.? is 6s. 9d.? is 4s. 8d.?

Questions can be asked in the other currencies in the same manner.

REDUCTION OF CURRENCIES TO FEDERAL MONEY.

Sums of this kind, which are too complicated to be done mentally, may be performed on the slate, by the following rules.

TO REDUCE BRITISH CURRENCY TO FEDERAL MONEY.

Reduce the sum to a decimal of the pound order, and divide the answer by $\frac{9}{40}$.

The reason of this rule is, that a dollar is $\frac{9}{40}$ of a £ of this currency, and therefore there are as many dollars in the sum as there are $\frac{9}{40}$ in it.

NOTE. Before reducing *any* currency to Federal money, the sum must *be reduced to a decimal of the pound order*. After this process the following rules may be used.

TO REDUCE CANADA CURRENCY.

As a dollar is $\frac{1}{4}$ of a £ in this currency, there will be as many dollars as there are $\frac{1}{4}$ in the sum. Therefore

Reduce the sum to the decimal of a £ and divide it by $\frac{1}{4}$.

TO REDUCE NEW ENGLAND CURRENCY.

As 1 dollar is ,3 of a pound in this currency, so there are as many dollars in a sum of N. England currency as there are ,3 in it. Therefore

Reduce the sum to the decimal of a £ and divide it by ,3.

TO REDUCE NEW YORK CURRENCY.

As 1 dollar is ,4 of a pound in this currency, there will be as many dollars in a sum of New York currency, as there are ,4 in it. Therefore

Reduce the sum to the decimal of a £ and divide it by ,4

What is the rule to reduce British currency to Federal money? Canada? New England? New York?

TO REDUCE PENNSYLVANIA CURRENCY.

As 1 dollar is $\frac{3}{8}$ of a £ in this currency there are as many dollars in the sum as there $\frac{3}{8}$ contained in it. Therefore

Reduce the sum to the decimal of a £ and divide it by $\frac{3}{8}$.

TO REDUCE GEORGIA CURRENCY.

As 1 dollar is $\frac{7}{30}$ of a pound in this currency, there are as many dollars in the sum as there are $\frac{7}{30}$ contained in it. Therefore,

Reduce to the decimal of a £ and divide the sum by $\frac{7}{30}$.

REDUCTION OF FEDERAL MONEY TO THE SEVERAL CURRENCIES.

To change a sum in Federal money to the different currencies, the preceding process is reversed, and the sum is to be *multiplied (instead of divided)* by the several fractions. The answer is found in pounds and decimals of a pound. The decimal can be reduced to units of the shilling and pence order by a previous rule. (p. 158.)

EXAMPLES.

1. Reduce 1s. 6d. in the several currencies to Federal money.

	Answers.
Of Canada Currency, it is	\$,30
British, "	\$,333$\frac{1}{3}$
N. England, "	\$,25
N. York, "	\$,187$\frac{1}{2}$
Penn. "	\$,20
Georgia. "	\$,321$\frac{3}{7}$

2. Reduce 4$\frac{1}{2}$d. of the several currencies to Federal money.
3. Reduce 4s. 6d. of the several currencies to Federal money.
4. Reduce 35£ 3s. 7$\frac{1}{2}$d. of the several currencies to Federal money.
5. Reduce \$ 118,25 to the several currencies.

Pennsylvania? Georgia? What is the rule for the reduction of Federal money to the several currencies?

Answers to the last.

	£	s.	d.
In Canada currency, it is	29 "	11 "	3
British, "	26 "	12 "	1½
N. Eng. "	35 "	9 "	6
N. York, "	47 "	6 "	0
Penn. "	44 "	6 "	10½
Georgia, "	27 "	11 "	9¾

Reduce 2s. 9d. of N. England currency to the same value in all other currencies.

Reduce 4s. 6d. N. York currency to the same value in all the other currencies.

REDUCTION FROM ONE CURRENCY TO ANOTHER.

The following table will enable the pupil to reduce a sum from one currency to another, with more facility than by any other method. Each fractional figure shows the relative value of a sum in one currency to the same sum in another currency

For example, the ¾ in the *second* perpendicular and the *fourth* horizontal column, shows that £1 sterling is ¾ of the number which expresses the same value in New England currency. Thus £6 sterling is ¾ of the number which expresses the same value in New England currency. That is, £6 is ¾ of the answer to be obtained when the same value is expressed in New England currency. To find the answer, we reason thus. If £6 is *three* fourths, £2 is *one* fourth, and £8 is the answer: thus dividing by ¾.

RULE FOR CHANGING A SUM IN ONE CURRENCY, TO THE SAME VALUE IN ANOTHER CURRENCY.

To change a sum in a currency written in the upper space to one written in the right hand space, divide by the fraction that stands where both spaces meet.

If there are shillings, pence and farthings in the sum, first reduce them to the decimal of a £.

What is the rule for the reduction of one currency to another?

TABLE

EXHIBITING THE COMPARATIVE VALUES OF THE SEVERAL CURRENCIES.

ANY SUM EXPRESSED IN										
$ Fed. M.	£ Ster.	£ Geor.	£ Irish.	£ Can.	£ N. E.	£ Penn.	£ N. Y.	£ Scotch	IS	OF THE SAME SUM EXPRESSED IN
$\frac{10}{27}$	$\frac{1}{12}$	$\frac{7}{81}$	$\frac{250}{2769}$	$\frac{5}{54}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{4}{27}$		£ Scot.	
$\frac{10}{4}$	$\frac{9}{16}$	$\frac{7}{12}$	$\frac{1125}{1846}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{16}{15}$		$\frac{27}{4}$	£ N. Y.	
$\frac{8}{3}$	$\frac{3}{5}$	$\frac{28}{45}$	$\frac{600}{923}$	$\frac{2}{3}$	$\frac{4}{5}$		$\frac{16}{15}$	$\frac{36}{5}$	£ Pen.	
$\frac{10}{3}$	$\frac{3}{4}$	$\frac{7}{9}$	$\frac{750}{923}$	$\frac{5}{6}$		$\frac{5}{4}$	$\frac{4}{3}$	9 times	£ N. E.	
4 times	$\frac{9}{10}$	$\frac{14}{15}$	$\frac{900}{923}$		$\frac{6}{5}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{54}{5}$	£ Can.	
$\frac{923}{225}$	$\frac{923}{1000}$	$\frac{6461}{6750}$		$\frac{923}{900}$	$\frac{923}{750}$	$\frac{923}{600}$	$\frac{1846}{1125}$	$\frac{2769}{250}$	£ Irish.	
$\frac{30}{7}$	$\frac{27}{28}$		$\frac{6750}{6461}$	$\frac{15}{14}$	$\frac{9}{7}$	$\frac{45}{28}$	$\frac{12}{7}$	$\frac{81}{7}$	£ Geo.	
$\frac{40}{9}$		$\frac{28}{27}$	$\frac{1000}{923}$	$\frac{16}{9}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{16}{9}$	12 times	£ Ster.	
	$\frac{9}{40}$	$\frac{7}{30}$	$\frac{225}{923}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{3}{8}$	$\frac{4}{10}$	$\frac{27}{10}$	£ F. M.	

EXAMPLES FOR PRACTICE.

1. Reduce £4 N. E. to F. M. *Ans.* $\$13{,}333\frac{1}{3}$.
2. Reduce £2 3s. 9d. N. E. to F. M. *Ans.* $\$7{,}291\frac{2}{3}$.
3. Reduce £6 N. Y. to F. M. *Ans.* $15.00.
4. Reduce £8; 4; 9 N. Y. to F. M. *Ans.* $\$20{,}593\frac{3}{4}$.
5. Reduce £3; 2; 3 Penn. to F. M. *Ans.* $8.30.
6. Reduce $152.60 to N. E. *Ans.* £45; 15; 7.2.
7. Reduce $196.00 to N. E. *Ans.* £58; 16.
8. Reduce $629.00 to N. Y. *Ans.* £251; 12.
9. Reduce £35; 6; 8 sterling to N. E. *Ans.* £47; 2; 2; $2\frac{2}{3}$.
10. Reduce £120 N. E. to Can. *Ans.* $100.
11. Reduce £155; 13 N. E. to Sterling. *Ans.* £116; 14; 9.
12. Reduce £104; 10 Can. to N. Y. *Ans.* £167; 4.

13. Reduce £300; 10; 4; 2 Can. to Penn.
Ans. £450; 15; 6; 3.
14. Reduce £937; 18; 11; 1 N. E. to Geo.
Ans. £721; 14; 8. 3.
15. Reduce $224; 60 to Can. *Ans.* £56; 3.
16. Reduce £225; 6 N. E. to F. M. *Ans.* $752.00
17. Reduce £880 15; 11; 1 Penn. to Sterling.
Ans. 528 9; 6; 3.
18. Reduce £6,750 Irish to Geor. *Ans.* £6,461.
19. Reduce £1,846 Ster. to Irish. *Ans.* £2,000.
20. Reduce £1,722; 18; 9; 3 N. E. to N. Y.
Ans. £2,298; 5; 1.
21. Reduce £2,114; 1; 3 Can. to F. M. *Ans.* $8,456.25.
22. Change £784; 5; 6; 2 Penn. to Geor.
Ans. £487; 19; 10; $2\frac{32}{45}$.
23. Change £923 Sterling to Irish.
24. Change £4,000 Irish to Sterling.
25. Change £157; 8; 3; 3 N. Y. to N. E.
26. Change £1,654; 3; 8; 1 Penn. to N. E.
27. Change £947; 9; 4; 2 N. E. to F. M.
28. Change $1,444.66 to N. E. To N. Y. To Penn.
29. Change $945.22 to N. Y. To Geor. To Can.
30. Change £1,846; 15; 4 N. E. to F. M. To Penn. To Georgia.
31. Change $$4,444,444\frac{4}{9}$ to Sterling.
32. Reduce £1,000,000 Sterling to F. M.

ARITHMETIC.

THIRD PART.

NUMERATION.

In the following, *Third Part*, there will be a review of the preceding subjects, embracing the more difficult operations. The rules and explanations will not be repeated, as the pupils can refer to them in the former part.

ROMAN NUMERATION.

Before the introduction of the Arabic figures, a method of expressing numbers by *Roman Letters* was employed. As this method has not entirely gone out of use, it is important that it should be learned. The following letters are employed to express numbers.

I. One.	X. Ten.
II. Two.	L. Fifty.
III. Three.	C. One Hundred.
IIII. or IV. Four.	D. Five Hundred.
V. Five.	M. One Thousand.

The above letters, by various *combinations*, are made to express all the numbers ever employed in Roman Numeration.

RULE FOR WRITING AND READING ROMAN NUMBERS.

As often as a letter is repeated, its value is repeated. When a less number is put before a greater, the less number is subtracted. But when the less number is put after the greater, it is added to the greater.

Examples. In IV. the less number, I. is put *before* the greater number V. and is to be *subtracted*, making the number *four*.

What is Roman numeration? What is the rule for writing and reading Roman numbers?

In VI. the less number is put *after* the greater, and it is to be *added*, making the number *six*.

In XL the ten is *subtracted* from the fifty.

In LX the ten is *added* to the fifty.

The following is a table of Roman Numeration.

TABLE.

One	I	Ninety	LXXXX or XC
Two	II	One hundred	C
Three	III	Two hundred	CC
Four	IIII or IV	Three hundred	CCC
Five	V	Four hundred	CCCC
Six	VI	Five hundred	D or IↃ *
Seven	VII	Six hundred	DC
Eight	VIII	Seven hundred	DCC
Nine	VIIII or IX	Eight hundred	DCCC
Ten	X	Nine hundred	DCCCC
Twenty	XX	One thousand	M or CIↃ †
Thirty	XXX	Five thousand	IↃↃ or $\overline{V}$ ‡
Forty	XXXX or XL	Ten thousand	CCIↃↃ or $\overline{X}$
Fifty	L	Fifty thousand	IↃↃↃ
Sixty	LX	Hundred thousand	CCCIↃↃↃ or $\overline{C}$
Seventy	LXX	One million	$\overline{M}$
Eighty	LXXX	Two million	$\overline{MM}$

* IↃ is used instead of D. to represent five hundred, and for every additional Ↄ annexed at the right hand, the number is increased *ten times*.

† CIↃ is used to represent one thousand, and for every C and Ↄ put at each end, the number is increased *ten times*.

‡ A line over any number increases its value *one thousand times*.

Write the following numbers in Roman letters:

5. 7. 3. 9. 8. 16. 4. 14. 5. 15. 6. 16. 26. 36. 306. 1. 11. 111. 7. 17. 77. 777. 1800. 1832. 1789.

Read the following Roman numbers:

VI. XIX. XXIV. XXXVI. XXIX. LV. XLI. LXIV. LXXXVIII. XCIX. MDCCCXVIII.

OF OTHER METHODS OF NUMERATION.

By the common method of numeration, *ten* units of one order make one unit of the next higher order. But it is equally practicable, to have any other number than ten, to constitute a unit of a higher order. Thus we might have *six* units of one order to make one unit of the next higher order. Or *twelve* units of one order might make one of the next higher order.

The number which is selected to constitute units of the higher orders, is called the *radix* of that system of numeration.

The *radix* of the common system is *ten*, and this number, it is supposed, was selected, because men have ten fingers on their hands, and probably used them in expressing numbers.

Before the introduction of the Arabic figures, Ptolemy introduced a method of numeration, in which *sixty* was the *radix*. The Chinese and East Indians use it to this day.

But in Ptolemy's system there were not sixty *different characters* employed. Instead of this, the Roman method of numeration was used for all numbers as far as *sixty*, and then for the next higher orders the *same letters* were used over again, with an accent (′) placed at the right. For the *third* order two accents (″) were used, and for the *fourth* order three accents (‴).

To illustrate this method by Arabic figures, 31′ 23 signifies 31 *sixties* and 23.

We have some remnants of this method in the division of time into 60 seconds for a minute, and 60 minutes for an hour, and also the division of the degrees of a circle, into 60 seconds to a minute, and 60 minutes to a degree.

EXERCISES IN NUMERATION, COMMON, VULGAR, AND DECIMAL.

(*See rules on pages* 46, 51, *and* 57.)

1. Two million, four thousand, one hundred and six.
2. Two hundred thousand, and six *tenths*.
3. Twenty-six billion, six thousand, and fifteen *thousandths*.
4. Two hundred and sixty thousand *millionths*.
5. One *sixth* of *two apples* are how much, and how written?
6. One *ninth* of *twenty oranges* are how much, and how written? Is it a proper or improper fraction?
7. One *sixth* of *four bushels* is how much? how written? is it a proper, or improper fraction?
8. One tenth of forty bushels, how much? how written? is it a proper or improper fraction?
9. One *tenth* of *three oranges*, how much? how expressed?
10. Three *tenths* of *three oranges*, how much? how expressed?
11. Four *sixths* of *twelve apples*, how much? how expressed?

What other methods of numeration are there? What is the *radix*? What is the radix of the *common* system? Of Ptolemy's?

12. Three thousand *tenths of thousandths.*

13. Four billions, six thousand, and five *ten thousandths.*

14. Sixteen billions, three hundred and six millions, five hundred thousand, and six *tenths of millionths.*

15. Five trillion, five million, five units, and three hundred and sixty-five *millionths.*

16. Sixteen hundred and twenty-four, and four *tenths of billionths.*

ADDITION.

Let the pupil add the following numbers:

1

Two hundred and six million; twenty-four thousand, five hundred and six.

Thirty-seven billion, twenty-six thousand and three.

Four hundred and seventy-nine billion, six hundred and sixty-seven million, nine hundred and eighty-four thousand, six hundred and ninety-nine.

Fifteen million, seventy-seven thousand, nine hundred.

Thirty-six trillion, four hundred million, and six.

Four quadrillion, seventeen million, three hundred and six.

Six quadrillion, fourteen trillion, seventeen million, fourteen thousand, three hundred and nine.

Twenty-four sextillion, five hundred million and nine.

2

Sixteen thousand, four hundred and sixty-four, and nine *tenths.*

Two hundred and sixty-nine million, fourteen hundred and three, and thirteen *hundredths.*

Forty-four million, three thousand and six, and twenty *thousandths.*

Five hundred million, nine hundred and ninety-nine thousand, eight hundred and seventy-nine, and two hundred and sixty-four *tenths of thousandths.*

Six hundred and seventeen thousand, four hundred and sixty-eight, and five hundred and seventy-nine *hundredths of thousandths.*

Forty-six million, nine thousand, and seventy *millionths.*

3

Add two twelfths, three *fourths*, and four *sixths*. (See p. 150.)

4

Add twenty-four *fiftieths*, sixteen *tenths*, and twenty *halves*.

5

Add forty-nine *eightieths*, seventy-nine *fortieths*, and two hundred *thousandths*.

6

Add nine *twenty-sevenths*, thirteen *forty-fourths*, and twenty-nine *seventieths*.

SUBTRACTION.

1

From three hundred and sixty-nine million, four hundred twenty-seven thousand, three hundred seventy-six, subtract two hundred and ninety-three million, four hundred and eighty-three thousand, nine hundred and eighty-seven.

2

From twenty-four billion, six hundred and thirteen million, four hundred and forty-four thousand, eight hundred and eighty-six, and twenty-nine *hundredths*, subtract sixteen billions, twenty-four thousand and sixteen, and four hundred and six *thousandths*.

3

From sixty-four sextillion, ninety trillion, seven billion, twenty-nine million, forty thousand three hundred and six, and twenty-nine *tenths of millionths*, subtract fourteen quintillions, nine quadrillions, seven trillions, fourteen thousand and eighty, and seven *hundredths of millionths*.

4

From nine *twelfths*, subtract two *fifths*. (See p. 150.)

5

From thirteen *twenty-sevenths*, subtract three *twenty-fourths*.

6

From three *fifths*, subtract twenty-nine *seventy-sevenths*.

7

From twelve hundred and six, *four hundred and twentieths*, subtract four hundred and nine, *nine hundred and ninetieths*.

MULTIPLICATION.

1. Multiply 32694302 by 365.
2. Multiply 24,2 by 27 (see p. 133.)
3. Multiply 324,92 by 236.
4. Multiply 236,49 by 2,4.
5. Multiply 47,2935 by 2, 68432.
6. Multiply 876,24 by 32,94.
7. Multiply 14 yds. 3 qrs. 2 na. by 28.
8. Multiply 8 le. 2 m. 6 fur. 22 po. by 362.
9. Multiply 2 bu. 3 pk. 1 qr. 1 pt. by 172.
10. Multiply $\frac{2}{6}$ by 3 (see p. 101.)
11. Multiply $\frac{3}{7}$ by 48.
12. Multiply $\frac{17}{25}$ by 32.
13. Multiply 12 by $\frac{3}{8}$ (see p. 105.)
14. Multiply 24 by $\frac{3}{8}$.
15. Multiply 324 by $\frac{5}{12}$.
16. Multiply 2342 by $\frac{27}{324}$.
17. Multiply $\frac{3}{4}$ by $\frac{2}{3}$.
18. Multiply $\frac{2}{7}$ by $\frac{3}{8}$.
19. Multiply $\frac{5}{8}$ by $\frac{3}{5}$.
20. Multiply $\frac{6}{18}$ by $\frac{42}{58}$.
21. Multiply $\frac{326}{432}$ by $\frac{17}{30}$.

SUMS FOR MENTAL EXERCISE.

Multiply 5 and $\frac{2}{6}$ by $\frac{1}{4}$.

Let such sums be *stated* thus:

One fourth of 5 is 1 unit, and 1 remains.

This remaining 1 is changed to sixths and added to the $\frac{2}{6}$, making $\frac{8}{6}$.

One fourth of *one* sixth would be $\frac{1}{24}$, therefore one fourth of eight sixths is $\frac{8}{24}$.

In the above operation we find that one fourth of 5 is 1 and 1 remains. This remainder is changed to sixths and added to the fraction $\frac{2}{6}$, and then is divided by 4. The answer is 1 and $\frac{8}{24}$.

1. If a yard of muslin cost $2\frac{1}{2}$, what will $\frac{1}{2}$ a yard cost? What is $\frac{1}{2}$ of $2\frac{1}{2}$?

2. If a barrel of wine cost $10\frac{1}{2}$ dollars, what cost $\frac{1}{2}$ a barrel? What is $\frac{1}{2}$ of $10\frac{1}{2}$?

3. If 4 bushels of rye cost 8 dollars and $\frac{2}{5}$, what cost 2 bushels? What is $\frac{1}{2}$ of $8\frac{2}{5}$?

4. If you have $2\frac{1}{2}$ oranges, and give $\frac{1}{5}$ away, how much do you keep? What is $\frac{1}{5}$ of $2\frac{1}{2}$? What is $\frac{1}{6}$ of $8\frac{1}{5}$?

5. If 9 bushels of wheat cost $18\frac{3}{4}$ dollars, how much is that a bushel? What is $\frac{1}{9}$ of $18\frac{3}{4}$?

6. If 12 pieces of linen cost $16\frac{3}{5}$ dollars, how much is that by the piece?

7. If 8 gallons of brandy cost $14\frac{2}{5}$ dollars, how much is that a gallon?

8. If 8 yards of broadcloth cost $28\frac{3}{5}$ dollars, how much is that a yard?

9. How much would 4 yards cost?

10. If a man bought 8 barrels of cider for $25\frac{3}{5}$ dollars, how much is one barrel?

11. How much is 9 barrels?

12. If 12 yards of linen cambric cost $42\frac{4}{5}$ dollars, what would 7 yards cost?

13. If you have 12 dollars and $\frac{1}{3}$, and lose $3\frac{1}{2}$ times as much, how much do you lose?

We first multiply the $12\frac{1}{3}$ by 3 and then by $\frac{1}{2}$.

3 times 12 is 36, and 3 times $\frac{1}{3}$ is $\frac{3}{3}$ or 1, which, added to 36, is 37.

12 and $\frac{1}{3}$ multiplied by $\frac{1}{2}$ is 6 and $\frac{1}{6}$, which added to the 37 makes 43 and $\frac{1}{6}$.

14. Multiply 8 and $\frac{2}{5}$ by 4 and $\frac{1}{3}$.

In doing this sum, first multiply the 8 and then the fraction by 4, and add the products together. Then multiply the 8, and the fraction by $\frac{1}{3}$, and add these to the former products.

Thus 4 times 8 is 32. Four times $\frac{2}{5}$ is $\frac{8}{5}$, which is $1\frac{3}{5}$. This added to 32 is 33 and $\frac{3}{5}$.

One third of 8 is 2, and 2 remains. Add 2 to the 33 making 35. Change the remainder to fifths and add the $\frac{2}{5}$ making $\frac{12}{5}$. One third of *one fifth* would be $\frac{1}{15}$, therefore $\frac{1}{3}$ of $\frac{12}{5}$ is $\frac{12}{15}$, which added to 35 and $\frac{3}{5}$ makes 35 and $\frac{21}{15}$, which equals 36 and $\frac{6}{15}$

15. Multiply 5 and $\frac{4}{5}$ by 2 and $\frac{1}{4}$.

16. Multiply 12 and $\frac{3}{5}$ by 2 and $\frac{2}{3}$.

17. Multiply 9 and $\frac{4}{10}$ by 6 and $\frac{2}{5}$.

18. Multiply 7 and $\frac{3}{5}$ by 4 and $\frac{6}{8}$.

19. Multiply 11 and $\frac{5}{6}$ by 3 and $\frac{4}{7}$.

20. Multiply 8 and $\frac{3}{7}$ by 8 and $\frac{1}{5}$.

21. Multiply 10 and $\frac{2}{3}$ by 7 and $\frac{1}{7}$.

22. If you buy 9 and $\frac{4}{5}$ gallons of wine and return $2\frac{3}{4}$ times as much, how much do you return?

23. If one boy takes 12 apples and $\frac{4}{5}$, and another takes $5\frac{2}{5}$ times as many, how many does the last take?

24. If one room requires 12 and $\frac{3}{8}$ yards of carpeting, and another requires 3 and $\frac{4}{5}$ times as much, how much is required?

DIVISION.

1. Divide 9123648 by 79632.
2. Divide 246,2 by 23 (See p. 140.)
3. Divide 2394,609 by 235.
4. Divide 3246,9214 by 39.
5. Divide 32,4 by 9,4 (See p. 140.)
6. Divide 3294 by 2,79.
7. Divide 324,976 by 2,4 (See p. 140.)
8. Divide 329,42 by 3,24.
9. Divide 329021,4639 by 296,029.
10. Divide 112 £. 12s. 7d. 4 qrs. by 38.
11. Divide 29 yds. 2 qrs. 3 na. by 39.
12. Divide 2 m. 5 fur. 17 po. 3 yds. by 91.
13. Divide 12 by $\frac{3}{4}$ (See p. 114.)
14. Divide 128 by $\frac{2}{8}$.
15. Divide 418 by $\frac{8}{12}$.
16. Divide 324 by $\frac{7}{28}$.
17. Divide 3297 by $\frac{16}{48}$.
18. Divide $\frac{12}{36}$ by 6 (See p. 117.)
19. Divide $\frac{32}{54}$ by 16.
20. Divide $\frac{365}{498}$ by 27.
21. Divide $\frac{2987}{4963}$ by 361.
22. Divide $\frac{125}{27032}$ by 249.
23. Divide $\frac{2}{4}$ by $\frac{3}{7}$ (See p. 121.)
24. Divide $\frac{3}{12}$ by $\frac{4}{7}$.
25. Divide $\frac{3}{24}$ by $\frac{2}{8}$.
26. Divide $\frac{234}{426}$ by $\frac{113}{126}$.

Examples for mental exercise.

1. Divide $\frac{1}{2}$ by $\frac{1}{8}$. Divide $\frac{1}{2}$ by $\frac{1}{12}$ (See page 147.)
2. Divide $\frac{1}{3}$ by $\frac{1}{13}$. Divide $\frac{1}{4}$ by $\frac{2}{12}$.
3. Divide $\frac{2}{4}$ by $\frac{4}{13}$. Divide $\frac{1}{3}$ by $\frac{1}{8}$.
4. Divide $\frac{2}{3}$ by $\frac{4}{6}$. Divide $\frac{4}{8}$ by $\frac{1}{16}$.
5. Divide $\frac{4}{8}$ by $\frac{4}{16}$. Divide $\frac{8}{16}$ by $\frac{1}{8}$.
6. Divide $\frac{1}{12}$ by $\frac{2}{4}$. Divide $\frac{3}{6}$ by $\frac{2}{12}$.
7. How many times is $\frac{1}{4}$ contained in $\frac{9}{12}$?
8. How many times is $\frac{3}{5}$ contained in $\frac{7}{8}$?

9. How many times is $\frac{3}{7}$ contained in $\frac{2}{3}$?

10. How many times is $\frac{2}{6}$ contained in $\frac{5}{6}$?

11. If beef is $\frac{1}{6}$ of a dollar a pound, how much can be bought for $\frac{2}{3}$ of a dollar?

12. If a yard of muslin cost $\frac{1}{12}$ of a dollar, how much can be bought for $\frac{2}{3}$ of a dollar?

In case the divisor and dividend have whole numbers with the fractions, the whole numbers must be *reduced* also, with the fractions, to a common denominator.

Thus if we wish to find how many $\frac{2}{3}$ there are in 4 and $\frac{3}{4}$, we must change the 4 and $\frac{3}{4}$ to twelfths, and the $\frac{2}{3}$ to twelfths also, and then divide as before. Thus; 4 and $\frac{3}{4}$ is $\frac{57}{12}$, and $\frac{2}{3}$ is $\frac{8}{12}$.

In 57 twelfths, there are 7 times 8 twelfths, and *one* twelfth left over. This *one* twelfth, is one *eighth* of the divisor $\frac{8}{12}$.

The answer then is 7 and $\frac{1}{8}$. That is, $4\frac{3}{4}$ contains $\frac{2}{3}$, just 7 times and $\frac{1}{8}$ of another time.

Again; how often is $2\frac{2}{3}$ contained in $5\frac{3}{4}$? First, reduce the divisor and dividend to fractions of a common denominator.

$2\frac{2}{3}$ is $\frac{32}{12}$, and $5\frac{3}{4}$ is $\frac{69}{12}$.

Divide 69 twelfths by 32 twelfths, and the answer is 2 and 5 *twelfths* left over.

This 5 twelfths is 5 *thirty secondths* of the divisor. For *five twelfths* is $\frac{5}{32}$ of 32 twelfths.

1. How many times is $1\frac{3}{4}$ contained in $8\frac{2}{3}$?

2. How many times is $2\frac{3}{4}$ contained in $5\frac{4}{6}$?

3. How many times is $9\frac{2}{3}$ contained in $16\frac{4}{5}$?

4. If you distribute $13\frac{4}{5}$ lbs. of flour among a certain number of persons, and give $2\frac{5}{6}$ lbs. to each, to how many persons do you give?

5. If $4\frac{2}{5}$ bushels of wheat last a family one week, how long will $12\frac{4}{7}$ bushels last them?

6. If $5\frac{3}{7}$ tons of hay will keep a horse 6 months, how many horses will $10\frac{2}{3}$ tons keep during the same time?

7. If a cistern is filled in $3\frac{2}{5}$ of an hour, how many times will the cistern be filled in $12\frac{6}{8}$ hours?

8. If you distribute $18\frac{6}{8}$ dollars among the poor, and give $2\frac{4}{8}$ dollars to each person, to how many do you give?

At $3\frac{1}{5}$ dollars a lb. how many pounds of gum can be bought dollars?

10. How many times is $\frac{5}{8}$ contained in $2\frac{4}{9}$?

11. How many times is $5\frac{2}{3}$ contained in $8\frac{6}{7}$?

12. How many times is $2\frac{3}{5}$ contained in $14\frac{6}{8}$?

13. How many times is $3\frac{2}{4}$ contained in $7\frac{3}{8}$?

14. How many times is $5\frac{4}{6}$ contained in $12\frac{2}{6}$?

REDUCTION.

1. In 29 gallons how many quarts? (See p. 86.)
2. In 65 pints how many gallons?
3. In 2 £. 14s. 9d. 3 qrs., how many farthings?
4. In 923469 farthings, how many pounds, shillings, and pence?
5. Reduce $\frac{2}{7}$ to a decimal. (See p. 145.)
6. Reduce $\frac{27}{55}$ to a decimal?
7. Reduce $\frac{123}{240}$ to a decimal?
8. Reduce $\frac{2}{3}$ $\frac{7}{9}$ and $\frac{4}{12}$ to a common denominator. (See p. 150.)
9. Reduce $\frac{4}{18}$ $\frac{2}{13}$ $\frac{9}{21}$ to a common denominator.
10. Reduce $\frac{2}{18}$ $\frac{4}{25}$ $\frac{8}{40}$ to a common denominator.
11. Reduce $\frac{128}{3428}$ to its lowest terms. (See p. 152.)
12. Reduce $\frac{2496}{4832}$ to its lowest terms.
13. Reduce $\frac{48}{68}$ to its lowest terms.
14. Reduce $\frac{5}{8}$ of a guinea to the fraction of a pound. (See p. 154.)
15. Reduce $\frac{2}{19548}$ to the fraction of a foot.
16. Reduce $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{3}{8}$ of a pound to the fraction of a shilling.
17. Reduce $\frac{3}{8}$ of $\frac{5}{8}$ of 3 shillings to the fraction of a pound.
18. What is the value of $\frac{2}{5}$ of a ton in lbs.? (See page 154.)
19. How many ounces in $\frac{3}{4}$ of a lb. Apothecary's weight?
20. How many pints in $\frac{6}{14}$ of a bushel?
21. Reduce 8 oz. 6 pwts. to the fraction of a lb. Troy. (See p. 155.)
22. Reduce 4 days 16 hours to the fraction of a year.
23. Reduce 36 gals. 4 qts. to the decimal of a hogshead. (See p. 157.)
24. Reduce 11d. 3 qrs. to the decimal of a shilling.

What is the value of ,169432 of a ton? (See p. 158.)

25. What is the value of ,24694 of a £?

What is the value of ,396 of an hour?

26. Reduce 7s. 8d. of each of the different currencies to the same value in Federal money. (See p. 161.)

27. Reduce $ 6, 29 to the same value in each of the different currencies. (See p. 162.)

INTEREST.

In conducting business, men often find it necessary to *borrow money* of each other, and it is customary to pay those who lend, for the use of their money until it is returned.

The *sum of money* lent, is called the *principal.*

The *sum paid for the use* of money, is called *interest.*

Amount is the *principal* and *interest* added together.

Per annum signifies *by the year.*

It is customary to pay a certain sum for every *hundred* dollars, pounds, &c. Thus in New England *six* dollars a year is paid for the use of every hundred, and in New York *seven* dollars for every hundred that is borrowed. The expressions *six per cent.*, *seven per cent.*, &c. signify that *six* or *seven* dollars are paid for every hundred borrowed. *Per* signifies *for*, and *cent.* is the abbreviation of *centum*, the Latin word for *hundred. Rate per cent.*, then, signifies *rate by the hundred.* When a man borrows a sum of money, he gives to the one of whom he borrows a writing in this form:

$ 500 ,00. Hartford, April 1, 1832.

On demand I promise to pay D. F. Robinson or order, five hundred dollars with interest, value received. Samuel Jones.

This is called a *note,* and is said *to be on interest.*

In this case the borrower, Samuel Jones, is obligated to pay six dollars a year for each hundred dollars, till the $ 500 are returned.

In Connecticut the law does not permit men to receive any more than 6 per cent. interest; in New York it allows 7 per cent., and the rate by law varies in the different states. When the *rate per cent.* is not mentioned, it is always to be understood

In the rule called interest, what is meant by principal? interest? amount? per annum? per cent.?

that the interest is what is allowed by the laws of the state where the note is given.

Usury is taking more interest than the law allows.

Legal interest is the rate allowed by law.

In all notes on interest, if no particular *rate per cent.* is mentioned, it is always understood to be *legal interest* that is promised. In this work, 6 *per cent.* will be understood when no *rate per cent.* is mentioned.

Sometimes it occurs that when a man has borrowed a sum of money, after a time he wishes to pay a *part* of the debt.

In this case, when the payment is made, the note which was given to the lender is taken, and an *endorsement* is written on it, stating that such a part of the note was paid at a particular time. After this the borrower only pays interest for that part of the debt which remains unpaid.

Notes are given either with or without interest. If the words " with interest" are not written, a note is understood to be without interest. If a note is given without interest, promising to pay *at a certain time*, after that time has expired, the note draws interest from that time.

Notes are given sometimes, promising to pay the interest *annually*, but oftener the interest is not to be paid until the note is paid.

When interest is paid only upon the sum lent, it is called *simple interest.*

But when the yearly interest is added each year to the principal, and then interest is taken upon both principal and interest, it is called *compound interest.*

The laws of the several states forbid taking compound interest; but a man who has lent money, can collect the interest every year, and put it out at interest, and thus gain compound interest.

But when a man borrows, if the creditor does not collect the interest *every year*, he cannot be compelled to pay interest on the interest.

In calculating interest, the *rate per cent.* is *a certain number* of *hundredths* of the sum lent. Thus if 1 per cent. is paid for

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Legal interest? usury? endorsements? What is meant by simple interest? compound?

$ 100, it is $\frac{1}{100}$ part of the sum lent. If 6 per cent. is paid, it is the $\frac{6}{100}$ part of the sum lent.

For this reason all calculations in interest are sums in decimal multiplication. We divide by the denominator to find *one* hundredth, by means of the separatrix, and multiply by the numerator to find the required number of hundredths. For example, if we wish to find the interest of $ 263 for one year, at 6 per cent. we must obtain the $\frac{6}{100}$ part of the $ 263. This is done by dividing by the denominator 100, by means of a separatrix, and multiplying by the numerator 6. In this case the *multiplication* is done first.

```
 $ 263
     6
 ------
$ 15,78
```

The *rate per cent.* therefore, may always be written as a *decimal fraction* of the order of *hundredths.*

1 per cent. is written		,01
2 per cent.	"	,02
$\frac{1}{2}$ per cent.	"	,005
$\frac{1}{4}$ per cent.	"	,0025
$\frac{3}{4}$ per cent.	"	,0075

Write $2\frac{1}{2}$ per cent. as a decimal fraction.

2 per cent. is ,02, and $\frac{1}{2}$ per cent. is, 005. Ans. ,025.

Write 4 per cent. as a decimal fraction. —— $4\frac{1}{2}$ per cent. —— $4\frac{3}{4}$ per cent. —— 5 per cent. —— $7\frac{1}{4}$ per cent. —— 8 per cent. —— $8\frac{3}{4}$ per cent. —— 9 per cent. —— $9\frac{1}{2}$ per cent. —— 10 per cent. (10 per cent. is $\frac{10}{100}$; decimally, ,10.) —— $10\frac{1}{2}$ per cent. —— 11 per cent. $12\frac{1}{2}$ per cent. —— 15 per cent.

1. If the interest on $ 1, for 1 year, be 6 cents, what will be the interest on $ 17 for the same time?

It will be 17 times 6 cents, or 6 times 17, which is the same thing: —

```
$17
 ,06
----
1,02
```
Answer; that is, 1 dollar and 2 cents.

To find the interest on any sum for 1 year, it is evident we need only to multiply it by the *rate per cent.* written as a *decimal* fraction. The product will be the interest required.

What is the interest of $ 121 at $3\frac{3}{4}$ per cent.? at $2\frac{1}{4}$ per cent.? at $8\frac{1}{2}$ per cent.? at $9\frac{3}{4}$ per cent.? at $4\frac{1}{4}$ per cent.?

When we wish to obtain the interest for *several* years, we have only to *multiply the interest of one year by the number of years.*

EXAMPLES.

What is the interest of $214 for 4 years at 2½ per cent.? for 3 yrs.? for 9 yrs.? for 24 yrs.?

What is the interest of $364,41 for 8 yrs. at 6¼ per cent.?

What is the interest of $1000 for 120 yrs.? *Ans.* $7200.

It may often be needful to calculate the interest on a sum, for a less time than a year.

When this is needful, the following mode is the most simple and expeditious.

Let the interest be at 6 per cent., as that is the most common rate.

At 6 per cent. each dollar gains 6 cents a year, (or 12 mo.) 6 cents for 12 mo. is ½ a cent (or 5 mills) for 1 month.

As 30 days is called a month, in calculating interest, 5 mills a month, is 1 mill for every 6 days.

Interest at 6 per cent. then gains on *each dollar*,

$,06 a year. $,005 a month. $,001 for every 6 days, and ⅙ of a mill for each day.

Whenever therefore we wish to calculate the interest of any sum for less than a year, we can first calculate the interest on *one dollar* for the given time, calculating 5 mills for every month, 1 mill for every 6 days, and ⅙ of a mill for each odd day.

After finding the interest for one dollar we can multiply this interest by the number of dollars in the sum.

EXAMPLES.

What is the interest of $36 at 6 per cent. for 9 mo. 12 days? for 6 mo. 3 days? for 8 mo. 18 days?

NOTE.—The *fractions* of a mill had better be changed to *decimals*. Thus instead of writing 5½ mills we can write ,0055—5⅓ mills can be written, ,0053+. (The sign of addition is added to the last, because there are more decimal orders that may be added.)

What is the interest of $334 for 4 mo. 2 d.? for 9 mo. 6 d.? for 7 mo. 4 d.?

What is the interest of $826 for 2 d.? for 5 mo. 3 d.? for 16 d.? for 9 mo. 16 d.?

If it is wished to obtain the interest of any sum for *less* than a year, at *any other* than 6 per cent. the method is, to find the interest at 6 per cent. and then take such parts of it, as the rate mentioned, is parts of 6 per cent.

Thus if we wish to find the interest of $ 560 for 4 mo. 8 d. at 5 per cent., we first find the interest at 6 per cent. for that time, and then subtract ⅙ of the sum from itself. For the interest at 5 per cent. is ⅙ less than the interest at 6 per cent.

Thus if the rate is 3 per cent. we must take ½ of the interest at 6 per cent.

If it is 4 per cent. we must take 4/6 (or ⅔) of the interest at 6 per cent., &c.

What is the interest of $ 241,62 cents for 8 mo. 6 days, at 2 per cent.? at 3 per cent.? at 4 per cent.? at 9 per cent.? at 12½ per cent.? at 15 per cent.?

What is the interest of $ 54.81 for 18 mo. at 5 per cent.? *Ans.* 4.11.

What is the interest of $ 500 for 9 mo. 9 days, at 8 per cent.? *Ans.* $ 31.00.

What is the interest of $ 62.12 for 1 mo. 20 days, at 4 per cent.? *Ans.* $ 0.345.

What is the interest of $ 85 for 10 mo. 15 days, at 12½ per cent.? *Ans.* $ 9.295.

RULES FOR CALCULATING INTEREST.

To find the interest for years.

Multiply the sum by the rate per cent. as a decimal of the order of hundredths, and the interest for one year is found. Multiply this answer by the number of years.

To find the interest for months and days.

Calculate the interest on one dollar for the given time, thus; calculate 5 *mills for every month,* 1 *mill for every six days, and* ⅙ *of a mill for each odd day. Add these together and multiply the answer by the number of dollars and cents in the sum, pointing off decimals according to rule.*

If the rate is any other than 6 *per cent., calculate the interest at* 6 *per cent., and then add to, or subtract from the sum such parts of itself, as the rate per cent. is parts of* 6 *per cent.*

What is the rule for calculating interest for one year? for a number of years? for parts of a year?

EXAMPLES.

What is the interest of $ 116,08 for 11 mo. 19 days?

Ans. $ 6.73,2

of $ 200 for 8 mo. 4 d.?	8,132
of 0,85 for 19 mo.?	0,08
of 8,50 for 1 yr. 9 mo. 12 d.?	0,909
of 675 for 1 mo. 21 d.?	5,737
of 8673 for 10 d.?	14,455
of 0,73 for 10 mo.?	,036

Rule for Sterling Money.

When the principal is pounds, shillings, and pence, reduce the sum to the decimal of a £ (see page 157), *and proceed as in Federal money. The answer is in decimals of a £, and must be changed back to units* (see page 158).

What is the interest of £ 36; 9s. 6½d. for 1 yr.?

Ans. £ 2. 3s. 9¼d

What is the interest of £ 36; 10s. for 18 mo. 20 d.?

Ans. £ 3. 8s. 3¼d.

What is the interest of £ 95 for 9 mo.? *Ans.* £ 4. 5s. 6d.

Find the interest on £ 13; 3; 6 for 1 yr. A. 15s. 9½d.

Find the interest on £ 13; 15s. 3½d. for 1 yr. 6 mo.

A. £1; 4; 9¼d.

Find the interest on £75; 8; 4 for 5 yrs. 2 mo.

A. £23. 7s. 7d.

Find the interest on £174; 10; 6 for 3 yrs. 6 mo.

A. £36. 13s.

Find the interest on £325; 12; 3 for 5 yrs. A. £97. 13s. 8d.

Find the interest on £150; 16; 8 for 4 yrs. 7 mo.

A. £41. 9s. 7d.

VARIOUS EXERCISES IN INTEREST.

TO FIND THE PRINCIPAL, WHEN THE TIME, RATE AND AMOUNT ARE KNOWN.

If in 1 yr. 4 mo. the interest and principal on a sum at 6 per cent. amount to $ 61,02, what is the principal?

What is the rule for Sterling money?

We first find what will be the amount of *a dollar with its interest*, for the given time. This amounts to $ 1,08. Now as every dollar in the original sum gained 8 cents interest, there were as many dollars as there are $ 1,08 in $ 61,02. $ 56,50.

RULE. *Find the interest of $ 1 for the given time and add to it. Divide the sum given by this amount.*

EXAMPLES.

What principal at 8 per cent. will amount to $ 85,12 in 1 yr. 6 mo.? *Ans.* $ 76.

What principal at 6 per cent. will amount to $ 99,311 in 11 mo. 9 d.? - *Ans.* $ 94.

TO FIND THE PRINCIPAL, WHEN THE TIME, RATE AND INTEREST ARE KNOWN.

What sum put at interest at 6 per cent. will gain $ 10,50 in 1 yr. 4 mo.?

One dollar put at interest for that time, would gain $,08, and therefore it requires as many one dollars as there are $,08 in $ 10,50. *Ans.* $ 131,25.

RULE. *Find the interest of $ 1 for the given rate and time. Divide the interest given by this, and the quotient is the principal.*

EXAMPLES.

A man paid $ 4,52 interest at the rate of 6 per cent. at the end of 1 yr. 4 mo. what was the principal? Ans. $ 56,50.

A man received $ 20 for interest on a certain note at the end of 1 yr. at the rate of 6 per cent. what was the principal?

TO FIND THE RATE, WHEN THE PRINCIPAL, INTEREST, AND TIME ARE KNOWN.

If $ 3,78 is paid for using $ 54, 1 yr. 6 mo. what is the rate per cent.?

If this sum were at interest at *one* per cent. it would produce $,54.

As many times therefore as $,54 is contained in $ 3,78 *so much more* than 1 per cent. is the rate.

RULE. *Divide the given interest by what would be the interest of the same sum at 1 per cent.*

What is the rule to find the principal, when the time, rate, and amount are known? How when time, rate, and interest are known? How find the rate, when principal, interest, and time are known?

If $ 2,34 is paid for the use of $ 468 for 1 mo. what is the rate per cent.? Ans. 6 per cent.

At $ 46,80 for the use of $ 520 for 2 yrs. what is it per cent.? Ans. 4½ per cent.

TO FIND THE TIME, WHEN THE PRINCIPAL, RATE AND INTEREST ARE KNOWN.

What is the time required to gain $ 3,78 on $ 36, at 7 per cent.?

We first find what would be the interest on that sum for *one year*, at 7 per cent.

This would be $ 2,52. As many times as this sum is contained in the interest mentioned in the sum, *so much more time* than one year is required.

RULE. *Divide the interest given, by the interest which the principal would gain at the same rate, in one year.*

Paid $ 20 for the use of 600 at 8 per cent.; what was the time? Ans. 5 mo.

Paid $ 28,242 for the use of $ 217,25 at 4 per cent.; what was the time? Ans. 3 yrs. 3 mo.

ENDORSEMENTS.

In transacting business, it is often necessary to calculate interest upon notes, where partial payments have been made, and *endorsed* upon the note. For example, a man borrows $ 500, and gives his note, promising to repay it with interest.

Two years after, he pays $ 150, and has it endorsed. Then two years after, he pays $ 75, and has it endorsed. At the end of six years he is ready to settle the note, and the question is, how much interest he shall pay.

There are different modes established by the laws of different states on this subject.

The three following are the most common. The *first* is the one which was formerly most commonly used.

FIRST METHOD.

Find the AMOUNT *of the principal for the whole time.*
Find the AMOUNT *of each payment to the time of settlement.*

How find time, when principal, rate, and interest are known?

Add the AMOUNTS *of the payments, and subtract them from the* AMOUNT *of the principal.*

EXAMPLE.

On April 1st, 1825, I gave a note to A. B. promising to pay him $ 300 for value rec'd. and interest on the same at 6 per cent. till settlement.

Oct. 1, 1825, I paid $ 100. April 16, 1826, paid $ 50. Dec. 1, 1827, paid $ 120.

What do I owe on April 1st, 1828?

$ cts. m.		ys. mo. da.
300,00,0	principal dated April 1, 1825.	
54,00,0	interest up to April 1st, 1828.	3. 0. 0.
354,00,0	amount of principal.	
100,00,0	1st payment, Oct. 1, 1825.	
15,00,0	interest up to April 1st, 1828.	2. 6. 0.
115,00,0	amount of 1st payment.	
50,00,0	2nd payment, April 16th, 1826.	
5,87,5	interest up to April 1st, 1828.	1. 11. 15.
55,87,5	amount of second payment.	
120,00,0	3rd payment, Dec. 1st, 1827.	
2,40,0	interest up to April 1st, 1828.	0. 4. 0.
122,40,0	amount of 3rd payment.	
55,87,5	" 2nd payment.	
115,00,0	amount of 1st payment.	
293,27,5	total amount of payments.	
354,00,0	amount of principal.	
293,27,5	total amount of payments subtracted.	
A. 60,72,5	remains due April 1st, 1828.	

RULE IN MASSACHUSETTS.

Find the Amount of the Principal to the time when one payment, or several payments together, exceed the interest due. From this subtract the payments, and the remainder will be a new Principal. Proceed thus till the time of settlement.

EXAMPLES.

For value received, I promise to pay James Lawrence $ 116,666 with interest. JOHN SMITH.

$ 116,666. May 1st, 1822.

On this note were the following endorsements.

Dec. 25, 1822,	received	$16,666.
July 10, 1823,	"	$ 1,666.
Sept. 1, 1824,	"	$ 5,000.
June 14, 1825,	"	$33,333.
April 15, 1826,	"	$62,000.

What was due August 3, 1827? *Ans.* $23,775.

The first principal on interest from May 1, 1822,		$116,666
Interest to Dec. 25, 1822, time of the first payment (7 months 24 days),		4,549
Amount,		$121,215
Payment, Dec. 25, exceeding interest then due,		16,666
Remainder for a new principal,		104,549
Interest from Dec. 25, 1822, to June 14, 1825 (29 months, 19 days),		15,490
Amount,		$120,039
Payment, July 10, 1823, less than interest then due,	$1,666	
Payment, Sept. 1, 1824, less than interest then due,	5,000	
Payment June 14, 1825, exceeding interest then due,	33,333	
		$39,999
Remainder for a new principal (June 14, 1825),		80,040
Interest from June 14, 1825, to April 15, 1826 (10 months 1 day),		4,015
Amount,		84,055
Payment, April 15, 1825, exceeding interest then due,		62,000
Remainder for a new principal (April 15, 1826),		$22,055
Interest due Aug. 3, 2827, from April 15, 1826 (15 months 18 days),		1,720
Balance due Aug. 3, 1827,		$23,775

The rule now adopted in Connecticut, is founded on the principle that interest is to be paid *by the year*, so that if a man pays before a year is ended, he receives interest on all he pays, from the time he pays it, to the end of the year when the interest is due.

Rule in Connecticut.

If the payment be made at the end of a year or more, add the interest due on the whole sum, at this time, to the principal, and subtract the payment.

Whenever other payments are made, proceed in the same manner, calculating interest on the principal from the time of the last payment.

If payment is made before a year has elapsed (from the date of the note, or from the last payment), find the amount of the principal for one year. Find also the amount of the payment from the time of payment to the end of the year when the interest would be due, and subtract the latter from the former. If, however, a year extends beyond the time of settlement, find the amount up to that time, instead of for a year.

If any remainder after subtraction, be greater than the preceding principal, then the preceding principal is to be continued as the principal for the succeeding time instead of the remainder, and the difference to be regarded as so much unpaid interest.

Let interest on the following note be calculated by the *three different rules.*

A note for $20,000 is given July 1st, 1825.

1st payment,	January 1st, 1826,	$1400
2d. do.	January 1st, 1827,	2000
3d. do.	September 1st, 1827,	5000

Settlement January 1st, 1829.

What is due on the note?

	Answers.
By the common rule,	$14,908,00
By the Massachusetts rule	15,212,96
By the Connecticut rule	15,209,47

Let the following be calculated by the Connecticut rule.

$1000,00 Hartford, Jan. 4, 1826.

On demand I promise to pay James Lowell, or order, one thousand dollars with interest; value received.

Hiram Simpson.

On this note were the following endorsements.

Feb. 19, 1827,	received	$200.00
June 29, 1828,	"	500.00
Nov. 14, 1828,	"	260.00
Dec. 29, 1831,	"	25.00

What is the balance, June 14, 1832? *Answer* $204.49.

Find the balance due on the following note by the Massachusetts rule.

$500.00. Hartford, Feb. 1. 1820.

Value received I promise to pay A. B. or order, five hundred dollars with interest. SAMUEL JONES.

Endorsements.	May 1, 1820, received,		$40.00
	Nov. 14, 1820,	"	8.00
	April 1, 1821,	"	12.00
	May 1, 1821,	"	30.00

How much remains, Sept. 16, 1821? *Ans.* $455.57.

Find the balance due on the following note by the Connecticut rule.

For value received I promise to pay G. B. or order, eight hundred and seventy-five dollars, with interest. $875.00

Hartford, Jan. 10, 1821. SAMUEL JONES.

Endorsements.	Aug. 10, 1824, received		$260.00
	Dec. 16, 1825,	"	300.00
	March 1, 1826,	"	50.00
	July 1, 1827,	"	150.00

What was due Sept. 1, 1828? *Ans.* $474.95.

The three rules used above, are all considered as objectionable. By the first rule, when a man pays a part of his debt, his payments are not applied to discharging the interest, but entirely to lessening the principal. By this rule, if a man should borrow a sum and promise to pay it, with the interest, in twenty-five years, if he should simply pay what would be the yearly interest, and have it endorsed, at the end of 25 years the debt would be entirely extinguished. Whereas if he should wait till the end of the time agreed upon, he would have to pay the *original sum* borrowed, and the *yearly interest* upon it also.

The objection to the other two rules is, that the man who makes payments before the time of settlement, actually is obliged to pay more than one who pays nothing before that time. Thus the most punctual man is obliged to pay more than the negligent.

Compound Interest is the only method which will do exact justice to both creditor and debtor. For a man who lends

What is the *common* rule for calculating interest on notes when there are endorsements? the Massachusetts rule? the Connecticut rule? What are the objections to the three rules? What is the only method to do justice to the creditor?

money is fairly entitled to receive interest *at the end of each year;* and then by investing the interest in other stock, he can obtain compound interest. The borrower, therefore, who detains this yearly interest, ought, in justice, to pay what the creditor could gain, if the debtor were punctual.

COMPOUND INTEREST.

Compound Interest is an allowance made for the use of the sum lent, and also for the use of the *interest* when it is not paid.

RULE. *Calculate the Interest, and add it to the principal at the end of a year. Make the Amount a new principal for the next year, with which proceed as before, till the time of settlement.*

1. What is the compound interest of $256 for 3 years, at 6 per cent.?

$256	given sum, or first principal.	
,6		
15,36	interest,	to be added together.
256,00	principal,	
271,36	amount, or principal for 2d. year.	
,06		
16,2816	compound interest, 2d. year,	added together
271,36	principal, do.	
287,6416	amount, or principal for 3d. year.	
,06		
17,25846	compound interest, 3d. year,	added together
287,641	principal, do.	
304,899	amount.	
256	first principal subtracted.	
A. $48,899	compound interest for 3 years.	

2. At 6 per cent. what will be the compound interest, and what the amount, of $1 for 2 years? —— what the amount for 3 years? —— for 4 years? —— for 5 years? —— for 6 years? —— for seven years? —— for 8 years?

Ans. to the last, $1,593+

It is plain that the amount of $2 for any given time, will be 2 times as much as the amount of $1; the amount of $3 will be 3 time as much, &c.

What is compound interest? What is the rule for performing it?

Hence, we may form the amounts of $1 for several years, into a table of *multipliers* for finding the amount of *any sum* for the same time. The following

TABLE,

Shows the amount of $ 1, or 1 £, &c. for any number of years, not exceeding 24, at the rates of 5 and 6 per cent. compound interest.

Y'rs.	5 per cent.	6 per cent.	Y'rs.	5 per cent.	6 per cent.
1	1,05	1,06	13	1,88564+	2,13292+
2	1,1025	1,1236	14	1,97993+	2,26090+
3	1,15762+	1,19101+	15	2,07892+	2,39655+
4	1,21550+	1,26247+	16	2,18287+	2,54035+
5	1,27628+	1,33822+	17	2,29201+	2,69277+
6	1,34009+	1,41851+	18	2,40661+	2,85433+
7	1,40710+	1,50363+	19	2,52695+	3,02559+
8	1,47745+	1,59384+	20	2,65329+	3,20713+
9	1,55132+	1,68947+	21	2,78596+	3,39956+
10	1,62889+	1,79084+	22	2,92526+	3,60353+
11	1,71033+	1,89829+	23	3,07152+	3,81974+
12	1,79585+	2,01219+	24	3,22509+	4,04893+

Note 1. Four decimals in the above numbers will be sufficiently accurate for most operations.

Note 2. When there are months and days, you may first find the amount for the *years*, and on that amount cast the interest for the months and days; this added to the amount will give the answer.

3. What is the amount of $600,50 for 20 years, at 5 per cent. compound interest? —— at 6 per cent.?

$1 at 5 per cent. by the table, is $2,65329; therefore, 2,65329 × 600,50 = $1593,30+ Ans. at 5 per cent.; and 3,20713 × 600,50 = $1925,881+ Ans. at 6 per cent.

4. What is the amount of $40,20 at 6 per cent compound interest, for 4 years? —— for 10 years? —— for 18 years? —— for 12 years? —— for 3 years and 4 months? —— for 24 years, 6 months, and 18 days? Ans. to last, $168,137.

How is the table used?

DISCOUNT.

Discount is a deduction made from a debt, for paying it before it is due.

If, for example, I owe a man $300 two years hence, and am willing to pay him now, I ought to pay only *that sum, which, with its interest*, would *in two years*, amount to $300.

The question then is, what sum, together with its interest at 6 per cent., would, in two years, amount to $300?

Such operations are performed by the rule for *finding the principal*, when the time, rate, and amount are given, (see p. 182).

The sum, which, in the time mentioned, would, by the addition of its interest, amount to the sum which is due, is called the *present worth*.

What is the present worth of $834, payable in 1 yr. 7 mo. 6 days, discounting at the rate of 7 per cent.? Ans. $750.

What is the discount on $321,63, due 4 years hence, at 6 per cent.? Ans. $62,26.

What principal, at 8 per cent., in 1 yr. 6 mo. will amount to $85,12? Ans. $76.

What principal, at 6 per cent. in 11 mo. 9 d. will amount to $99,311? Ans. $94.

How much ready money must be paid for a note of $18, due 15 months hence, discounting at the rate of 6 per cent.?

Ans. $16,744.

STOCK, INSURANCE, COMMISSION, LOSS AND GAIN, DUTIES.

Stock is a name for money invested in banks, in trade, in insurance companies, or loaned to a national government for the purpose of receiving interest.

Persons who invest money thus, are called *stockholders*.

When stockholders can sell their right to stock for more than they paid, it is said that *stock has risen*, and when they cannot sell it for as much as they paid, it is said that *stock has fallen*.

What is discount? What is the rule for performing it? What is stock? When is stock said to have risen or fallen?

Stock is bought and sold in *shares*, of from $50 to $100 a share.

The *nominal value* of a share is the amount paid, when the stock was first created.

The *real value* is the sum for which a share *will sell.*

When stock sells for its *nominal value*, it is said to be *at par.*

When it sells for *more* than its nominal value, it is said to be *above par*, and when for *less* it is *below par.*

When stock is *above par*, it is said to be so much per cent. *advance.*

An *Insurance Company*, is a body of men, who in return for a certain compensation, promise to pay for the loss of property insured.

The written engagement they give is called a *Policy.*

The sum paid to them for insurance, is called *Premium.*

Commission is a certain sum paid to a person called a *correspondent*, *agent*, *factor*, or *broker*, for assisting in transacting business.

Loss and Gain refer to what is made or lost, by merchants, in their business.

The calculations relating to *stock*, *insurance*, *commission*, *loss and gain*, and *duties*, are performed by the rule for calculating interest, when the time is one year.

RULE. *Multiply the sum given, by the rate per cent. as a decimal.* (See p. 180.)

EXAMPLES.

STOCK.—1. What is the value of $350.00 of stock at 105 per cent., that is, at 5 per cent. advance? Ans. $367.50.

The rate here is 105 per cent.=105 *hundredths.* The question then is, what is 105 hundredths of 350; or, multiply 350 by 1.05.

2. What is the value of 35 hundred dollar shares of stock at ¾ per cent. advance? Rate 1.0075. Ans. $3,526.25.

3. At 112½ per cent., what must I pay for $7,564.00 of stock? Rate 1.125. Ans. $8,509.50.

4. What is the value of $615.75 of stock, at 30 per cent. advance? Ans. $800.475.

What is the *nominal* value of a share? What the *real* value? When is stock at par? When above par? When below par? What is an insurance? Policy? Premium? What is commission? Loss and gain? What is the rule for performing these processes?

5. What is the value of $7,650.00 of stock at 119½ per cent.? Ans. $9,141.75.

6. What is the value of $1,500.00 of stock at 110 per cent.? Ans. $1,650.00.

7. What is the value of $3700 bank stock at 95½ per cent., that is at 4½ per cent. below par? Ans. $3,533.50.

INSURANCE.—1. What premium must be paid for the insurance of a vessel and cargo, valued at $123,425.00, at 15½ per cent.?

15½ per cent. = .155, and the question is, what is .155 of 123,425. Ans. $19,130.875.

2. What must I pay annually for the insurance of a house worth $3,500.00, at 1¾ per cent.? Ans. $61.25.

3. What must be paid for the insurance of property, at 6 per cent., to the amount of $2,500.00? Ans. $150.00.

4. What insurance must be paid on $375,000.00, at 5 per cent.? Ans. $18,750.00.

5. What premium must be annually paid for the insurance of a house worth $10,650.00, at 3 per cent.; and a store worth $15,875.00 at 4 per cent. and out houses worth $3,846.00, at 5 per cent.?

6. What premium must be annually paid for the insurance of a factory worth $30,946.00, at 10 per cent.; and 7 dwelling houses worth $875.00 each, at 8 per cent.; and 3 grist mills, worth $1,930.00 apiece, at 7 per cent.; and one storing house, worth $9,859.00 at 6 per cent.? Also, what is the average rate of insurance on the whole?

7. If I pay $930.00 annually for insurance, at 5 per cent., what is the value of the property insured?

Here 930 is .05 of the answer; 930÷.05=$18,600 *Ans*.

PROFIT AND LOSS.—1. Sold a bale of goods at $735.00, by which I gain at the rate of 6 per cent. What sum do I gain? Ans. $44.10.

2. In selling 50 hhds. of molasses, at 38 dollars a hhd., I gain 10 per cent. What is my gain? Ans. $190.00.

3. In selling 25 bales of cloth, each containing 27 pieces, and each piece 50 yards, a merchant gained 20 per cent. on the cost, which was 10 dollars a yard. What did he gain, and what did he sell the whole for?

Ans. Gain, $67,500.00. Whole, $405,000.00.

4. A merchant gained at the rate of 15 per cent. in selling the following articles: 6 hhds. of brandy for which he paid

$1.50 per gal.; 7 barrels of flour, cost 11 dollars a barrel; 2 quintals of fish, cost 4 cents a pound; 16 hhds. of molasses, cost 56 cents per gal. and 25 bls. of sugar, containing each 175 lbs., cost 9 cents per lb. What was his gain on the whole, and what did he receive in all?

COMMISSION.—1. If my agent sells goods to the amount of $2,317.46, what is his commission at $3\frac{1}{4}$ per cent.?

Ans. $75,31745.

2. What commission must be allowed for a purchase of goods to the amount of $1,286.00, at $2\frac{1}{2}$ per cent.? Ans. $32.15.

3. What commission shall I allow my correspondent for buying and selling on my account, to the amount of $2,836.23, at 3 per cent.?

4. A merchant paid his correspondent $25.00 commission on sales to the amount of $1,250.00. At what per cent. was the commission?

He paid him $\frac{25}{1250}=\frac{1}{50}=\frac{2}{100}=.02=2$ per cent. *Ans.*

DUTIES.—*Duty* is a certain sum paid to government for articles imported.

When duty is at a certain rate *on the value*, it is said to be *ad valorem*, in distinction from duties imposed *on the quantity*.

An Invoice is a written account of articles sent to a purchaser, factor, or consignee.

In computing duties, *ad valorem*, (or *ad val.* as it is commonly written,) it is usual in custom houses to add one tenth to the invoice value, before casting the duty. This makes the *real* duty one tenth greater than the *nominal* duty. It will be equally well to make the *rate* one tenth greater, instead of increasing the invoice.

1. Find the duty on a quantity of tea, of which the invoice is $215.17, at 50 per cent. Ans. $118.3435=$$118.343\frac{1}{2}$.

In this example we may add, as directed above, one tenth of 215.17, to 215.17. Thus, 215.17+$21.517=236.687. Then 236.687×50=$118.3435. Or we may add to the rate .50, one tenth of itself=.05: thus, .50+.05=55. Then, 215.17×.55=$118.3435, as before.

2. Find the duty on a quantity of hemp at $13\frac{1}{2}$ per cent., of which the invoice is $654.59. The second of the above modes is recommended. Another might be used, viz.: to find, first,

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What is duty? When are duties *ad valorem*? What is an invoice?

the duty on the invoice at the given rate, and add to it one-tenth of itself. Thus, 654.59 × 13½ = $88.36965. Ans. $97.206615.

3. What is the duty on a quantity of books, of which the invoice is $1,670.33, at 20 per cent.? Ans. $367.4726.

EQUATION OF PAYMENTS.

Equation of payments is a method of finding a time for paying several debts due *at different times*, all at once; and in such a way that both creditor and debtor will have the same value, as if the debts were paid at the several times promised.

For if a man pays a debt *before* it is due, the creditor gains; if he pays it *after* it is due, the debtor gains.

In how many months will $1 gain as much at interest as $8 will gain in 4 months? Now as the $1 is 8 times *less* than 8, it will require 8 times *more* time, or 8 × 4 = 32 months.

In how many months will the interest on $9 equal the interest on $1 for 40 months?

Supposing a man owes me $12 in 3 months, $18 in 4 months, and $20 in 9 months. He wishes to pay the whole at once; in what time ought he to pay?

$12 for 3 months = $1 for 36 months.
$18 for 4 months = $1 for 72 months.
$20 for 9 months = $1 for 180 months.
$50   288 months.

Now it appears that it will be the same to him to have $1 for 36, for 72, and for 180 months, as it would to have the 12, the 18, and the 20 dollars for the number of months specified.

He might therefore keep $1 just 288 months, and it would be the same as keeping the $50 for the number of months specified. But as the whole sum of money lent was $50, he may keep this only *one fiftieth* ($\frac{1}{50}$) of the time he might keep $1. Therefore divide the 288 months by the 50, and the answer is $5\frac{38}{50}$ months.

RULE FOR FINDING THE MEAN TIME OF SEVERAL PAYMENTS.

Multiply each sum by the time of its payment. Divide the sum of these products by the sum of the payments, and the quotient is the mean time.

What is Equation of payments? What is the rule for performing it?

A man is to receive $500 in 2 mo.; $100 in 5 mo.; $300 in 4 mo. If it is paid all at once, at what time should the payment be made?

A man owes me $300, to be paid as follows: $\frac{1}{3}$ in 3 months; $\frac{1}{3}$ in 4 months; and the rest in 6 months; what is the mean time for payment? Ans. $4\frac{1}{3}$ months.

RATIO.

The word *ratio* means *relation;* and when it is asked what ratio one number has to another, it means *in what relation* does *one number stand to another.*

Thus, when we say the ratio of 1 to 2 is $\frac{1}{2}$, we mean that the relation in which 1 stands to 2, is that of *one half* to a *whole.*

Again, the ratio of 3 to 4 is $\frac{3}{4}$, that is, 3 is $\frac{3}{4}$ of 4, or stands in the relation of $\frac{3}{4}$ to the 4. So also the ratio of 4 to 3 is $\frac{4}{3}$; for the 4 is 4 *thirds* of 3, and stands to it therefore in the relation of $\frac{4}{3}$.

What is the relation of 11 to 12? of 12 to 11?

When therefore we find the ratio of one number to another, we find *what part of one number another is.*

Then the ratio of 4 to 6 is $\frac{4}{6}$; that is, 4 is 4 sixths of 6.

The ratio of one number to another, then, may always be expressed by a fraction in which the *first number*, (called the *antecedent*) is put for *numerator*, and the *second number* (called the *consequent*) is put for *denominator*. Thus the ratio of 8 to 4 is $\frac{8}{4}$. This is an *improper fraction*, and, changed to whole numbers, is 2 *units*. The ratio of 8 to 4, then, is 2. That is, 8 is twice 4, or stands to 4 in the relation of a *duplicate* or *double.**

* The pupil needs to be forewarned that there is a difference between French and English mathematicians in expressing ratio.

The French place the *antecedent* as *denominator*, and the *consequent* as *numerator*. The English, on the contrary, place the *antecedent* as *numerator*, and the *consequent* as *denominator*. It seems desirable that there should be an agreement on this subject, in school books at least. Two of the most popular Arithmetics now in use, have adopted the French method, viz. Colburn and Adams. It seems needful to mention this, that pupils may not be needlessly perplexed, if called upon to use different books.

The method used here, is the English; as the *most common*, and as *most*

What is ratio? How is the ratio between numbers expressed?

PROPORTION.

When quantities have the *same ratio*, they are said to be *proportional* to each other. Thus the ratio of 2 to 4 is $\frac{1}{2}$, and the ratio of 4 to 8 is $\frac{1}{2}$; that is, 1 has the same relation to 2, that 4 has to 8, and therefore these numbers are called *proportionals*. Again, 4 is the same *portion* or *part* of 8, as 10 is of 20, and therefore these numbers are called *proportionals*. A *proportion*, then is a *combination of equal ratios*.

Points are used to indicate that there is a proportion between numbers. Thus 4 : 8 : : 9 : 18 is read thus; 4 has the same ratio to 8, that 9 has to 18. Or more briefly, 4 is to 8, as 9 to 18.

It will always be found to be the case in proportionals, that multiplying the *two antecedents* into the *two consequents*, produce the same product. Thus, 2 : 4 : : 6 : 12

Here let the consequent 4 be multiplied into the antecedent 6, and the product is 24; and let the antecedent 2 be multiplied into the consequent 12, and the product also is 24.

If then we have only *three* terms in a proportion, it is easy to find the fourth. For when we have multiplied one antecedent into one consequent, we know that the term left out is a number that, multiplied into the remaining term, would produce the same product.

Thus let one term be left out of this proportion.

8 : 4 : : 12 :

Here the *consequent* is gone from the last ratio. We multiply the antecedent 12 into the consequent 4, and the answer is 48. We now know that the term left out, is a number which, multiplied into the 8, would produce 48. This number is found by dividing 48 by 8, the answer is 6.

Whenever, therefore, a term is wanting to any proportion, it can be found by multiplying one of the antecedents by one of the consequents, and dividing the product by the remaining number.

What is the number left out in this proportion?

3 : 12 : : 24 :

consonant with perspicuity of language. For there seems to be no propriety in saying that the relation of 2 *to* 4 is 4-2. The ratio *between* these two numbers may be either 4-2 or 2-4, but the relation of 2 *to* 4, to use language *strictly*, can be nothing but 2-4.

What is proportion?

What is the number left out in this proportion?

9 : 8 : : 27 :

In a proportion, the two *middle terms* are called the *means*, and the first and last terms are called the *extremes*.

RULE FOR FINDING A FOURTH TERM IN A PROPORTION.

Multiply the means together, and divide the product by the remaining number.

It is on this principle, that what is commonly called the "Rule of Three," is constructed. By this process, we find a fourth term when *three* terms of a proportion are given.

Such sums as the following are done by this rule.

If 4 yards of broadcloth cost $ 12, what cost 9 yards?

Now the cost is in proportion to the number of yards; that is, the same ratio exists between the *number of yards*, as exists between the *cost* of each.

Thus,—as 4 yards is to 9 yards, so is the cost of 4 yards to the cost of 9 yards. The proportion, then, is expressed thus:

yds. yds. $
4 : 9 : : 12 :

Here the term wanting, is the cost of 9 yards; and if we multiply the means together, and divide by the 4, the answer is 27; which is the other term of the proportion, and is the cost of 9 yards.

Again, if a family of 10 persons spend 3 bushels of malt a week, how many bushels will serve at the same rate when the family consists of 30?

Now there is the same ratio between the number of bushels eaten, as between the numbers in the family. That is, as is the ratio of 10 to 30, so is the ratio of 3 to the number of bushels sought. Thus, 10 : 30 : : 3 :

RULE OF PROPORTION; OR RULE OF THREE.

When three numbers are given, place that one as third term, which is of the same kind as the answer sought. If the answer is to be greater than this third term, place the greatest of the remaining number as the second term, and the less number as first term. But if the answer is to be less, place the less number as second term, and the greater as first.

Having three terms of a proportion given, how can a fourth be found?

In either case, multiply the middle and third terms together, and divide the product by the first. The quotient is the answer, and is always of the same order as the third term.

NOTE.—This rule may be used both for common, compound, and decimal numbers. If the terms are compound, they must be reduced to units of the lowest order mentioned.

Many of the sums which follow will be better understood if performed by the mode of analysis, which has been explained and illustrated in a former part.

For example, we will take the first sum done by the rule of proportion.

If 4 yards of broadcloth cost $ 12, what cost 9 yards?

We reason thus,—If 4 yards cost $ 12, *one* yard must cost a *fourth* of $ 12. Therefore, *divide* $ 12 by 4, and we have the cost of *one* yard. *Multiply* this by 9, and we have the cost of 9 yards.

(It is usually best to *multiply first*, and then *divide*, and it has been shown that this is more convenient, and does not alter the answer.)

Let the following sums be done by the *Rule of Proportion*, and then explained by *analysis*.

1. If the wages of 15 weeks come to 64 dols. 19 cts. what is a year's wages at that rate? Ans. $ 222,52 cts. 5 m.

2. A man bought sheep at $ 1.11 per head, to the amount of $ 51.6; how many sheep did he buy? Ans. 46.

3. Bought 4 pieces of cloth, each piece containing 31 yds. at 16s. 6d. per yard, (New England currency,) what does the whole amount to in Federal money? Ans. $ 341.

When a tun of wine cost $ 140, what cost a quart?

Ans. 13 cts. $8\frac{8}{10}$ m.

4. A merchant agreed with his debtor, that if he would pay him down 65 cents on a dollar, he would give him up a note of hand of 249 dollars 88 cents. I demand what the debtor must pay for his note? Ans. $ 162.42 cts. 2 m.

5. If 12 horses eat 30 bushels of oats in a week, how many bushels will serve 45 horses the same time? Ans. $112\frac{1}{2}$ bushels.

6. Bought a piece of cloth for $ 48.27 cts. at $ 1.19 cts. per yard; how many yards did it contain? Ans. 40 yds. 2 qrs. $\frac{30}{100}$.

What is the Rule of Three? What is the method of doing the Rule of Three?

7. Bought 3 hhds. of sugar, each weighing 8 cwt. 1 qr. 12 lb. at $ 7.26 cts. per cwt.; what come they to? Ans. $ 182.1 ct. 8 m.

8. What is the price of 4 pieces of cloth, the first piece containing 21, the second 23, the third 24, and the fourth 27 yards, at $ 1.43 cts. a yard?

Ans. $ 135.85 cts. 21+23+24+27=95 yds.

9. Bought 3 hdds. of brandy, containing 61, 62, 62½ gals. at $ 1.38 cts. per gallon. I demand how much they amount to?

Ans. $ 255.99 cts

10. Suppose a gentleman's income is $ 1,836 a year, and he spends $ 3.49 cts. a day, one day with another, how much will he have saved at the year's end? Ans. $ 562.15 cts.

11. A merch't. bought 14 pipes of wine, and is allowed 6 months credit, but for ready money gets it 8 cents a gallon cheaper; how much did he save by paying ready money?

Ans. $ 141.12 cts.

12. Sold a ship for 537*l*. and I owned ⅜ of her; what was my part of the money? Ans. £ 201 7s. 6d.

13. If $\frac{5}{16}$ of a ship cost $ 718.25 cents, what is the whole worth?

5 : 781,25 : : 16 : $ 2500 Ans.

14. If I buy 54 yards of cloth for £ 31. 10s. what did it cost per Ell English? Ans. 14s. 7d.

15. Bought of Mr. Grocer 11 cwt. 3 qrs. of sugar, at $ 8,12 per cwt. and gave him James Paywell's note for £ 19 7s. (New England currency) the rest I pay in cash; tell me how many dollars will make up the balance. Ans. $ 30.91.

16. If a staff 5 feet long casts a shade on level ground 8 feet, what is the height of that steeple whose shade at the same time measure 181 feet? Ans. 113⅛ ft.

17. If a gentleman has an income of 300 English guineas a year, how much may he spend, one day with another, to lay up 500 dollars at the year's end? Ans. $ 2,46 cts. 5 m.

18. Bought 50 pieces of kerseys, each 34 Ells Flemish, at 8s. 4d. per Ell English; what did the whole cost? Ans. £ 425.

19. Bought 200 yards of cambric for £ 90. but being damaged, I am willing to lose £ 7. 10s. by the sale of it; what must I demand per Ell English? Ans. 10s. 3¾d.

20. How many pieces of Holland, each 20 Ells Flemish, may I have for £ 23 8s. at 6s. 6d. per Ell English? Ans. 6 pieces.

21. A merchant bought a bale of cloth containing 240 yds. at the rate of $ 7½ for 5 yards, and sold it again at the rate of $ 11¼

for 7 yards; did he gain or lose by the bargain, and how much?
Ans. He gained $25,71 cts. 4 m.+

22. Bought a pipe of wine for 84 dollars, and found it had leaked out 12 gallons; I sold the remainder at 12½ cents a pint; what did I gain or lose? Ans. I gained $30.

23. A gentleman bought 18 pipes of wine at 12s. 6d. (N. Jersey currency) per gallon; how many dollars will pay the purchase? Ans. $3780.

24. Bought a quantity of plate, weighing 15 lb. 11 oz. 13 pwt. 17 grs., how many dollars will pay for it at the rate of 12s. 7d. (New York currency,) per ounce? Ans. $301,50 cts. 2 $\frac{6}{10}$ m.

25. A factor bought a certain quantity of broadcloth and drugget, which together cost £81 per yard, the quantity of broadcloth was 50 yards, at 18s. per yard, and for every 5 yards of broadcloth he had 9 yards of drugget; I demand how many yards of drugget he had, and what it cost him per yard?
Ans. 90 yards, at 8s. per yard.

26. If I give 1 eagle, 2 dollars, 8 dimes, 2 cents and 5 mills, for 675 tops, how many tops will 19 mills buy? Ans. 1 top.

27. If 100 dollars gain 6 dollars interest in a year, how much will 49 dollars gain in the same time? Ans. $2,94 cts.

28. If 60 gallons of water, in one hour, fall into a cistern containing 300 gallons, and by a pipe in the cistern, 35 gallons run out in an hour; in what time will it be filled? Ans. in 12 hours.

29. A and B depart from the same place and travel the same road; but A goes 5 days before B, at the rate of 15 miles a day; B follows at the rate of 20 miles a day; what distance must he travel to overtake A? Ans. 300 miles.

COMPOUND PROPORTION.

Compound proportion, is a method of performing such operations in proportion, as require two or more statings. It is sometimes called *Double Rule of Three*, because its operations can be performed by two operations of the Rule of Three.

For example: If 56 lbs. of bread are sufficient for 7 men 14 days, how much bread will serve 21 men 3 days?

Here the amount of bread consumed depends upon two circumstances, the *number of days*, and the *number of men*.

We will first consider the quantity of bread as depending upon

the *number of men*, supposing the number of days to be the same.

The proportion would then be this;

7 men : 21 men : : 56 lbs. to the number of lbs. required.

Here we multiply the *means* together, and devide the answer by 7, and the answer is 168. That is, if the *time* was the same, viz. 14 days, the 21 men would eat 168 lbs. in that time.

We now make a second statement thus:

14 days: 3 days : : 168 lbs. : number of lbs. required.

The result of this statement is 36 lbs. which is the answer.

In performing this operation, let the pupil notice that in the first statement, the 56 was multiplied by the 21 and the answer divided by 7. This gives the same answer as would be given, did we *divide first*, and *then multiply*.

That is, 56 multiplied by 21, and the product divided by 7, is the same as 56 divided by 7 and the quotient multiplied by 21.

We divide by 7, to find how much *one* man would eat in the same time, or 14 days, and multiply by 21, to find how much 21 men would eat.

When we make the second statement, as we have found how much 21 men would eat in 14 days, we divided the quantity (168 lbs.) by 14, to find how much they would eat in *one* day, and then multiply by 3, to find how much they would eat in 3 days. But in this case also, the *multiplication* is done *first*.

Let the pupil also notice that the 56 lbs. was multiplied by 21 and divided by 7, and then that the *answer* to this (168 lbs.) was multiplied by 3 and divided by 14. Here 21 and 3 are used as *multipliers*, and 14 and 7 are used as *divisors*.

The answer will be the same (as may be found by trial) if 56 is multiplied by the *product of these multipliers*, and the answer divided by the *product of the divisors*.

It is on this principle that the common rule in compound proportion is constructed, which is as follows.

RULE OF COMPOUND PROPORTION.

Make the number which is of the same kind as the answer required, the THIRD *term.*

Take any two numbers of the same kind, and arrange them in regard to this third term, according to the rule of proportion. Then take any other two numbers of the same kind, and arrange them in like manner, and so on till all the numbers are used.

Then multiply the third term, by the product of the second terms,

What is compound proportion? What is the rule?

and divide the answer by the product of the first terms. The quotient is the answer.

EXAMPLES.

1. If a man travel 273 miles in 13 days, travelling only 7 hours a day, how many miles will he travel in 12 days at the rate of 10 hours a day?

Here the number, which is of the same kind as the answer required, is the 273 miles, and this is put as *third* term.

We now take two numbers of the same kind, viz. 13 days and 12 days, and placing them according to the rule of simple proportion, the question would stand thus.

$$13 : 12 : : 273$$

We next take two other numbers of the same kind, viz. 10 hours, and 7 hours, and arrange them *under* the former proportion according to the same rule, thus:

$$\left.\begin{matrix} 13 : 12 \\ 7 : 10 \end{matrix}\right\} : : 273$$

We now multiply the 273 by the *product* of 12 and 10, and divide by the *product* of 13 and 7, and the quotient is the answer.

We can explain this process analytically, thus:

We divide by 13, to find how much the man would travel in *one* day, at the rate of 7 hours per day.

We multiply by the 12, to find how much he would travel in 12 days, at the same rate.

We divide by 7 to find how much he would travel in *one* hour, and multiply by 10 to find how much he would travel in 10 hours.

Let the pupils explain the following in the same manner.

EXAMPLES.

2. If £100 in one year gain £5 interest, what will be the interest of £750 for 7 years? Ans. £262 10s.

3. What principal will gain £262 10s. in 7 years at 5 per cent. per annum? Ans. £750.

4. If a footman travel 130 miles in 3 days, when the days are 12 hours long; in how many days, of 10 hours each, may he travel 360 miles? Ans. $9\frac{63}{65}$ days.

5. If 120 bushels of corn can serve 14 horses 56 days, how many days will 94 bushels serve 6 horses? Ans $102\frac{16}{45}$ days.

6. If 7 oz. 5 pwts. of bread be bought at $4\frac{3}{4}$d. when corn is at 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price of the bushel is 5s. 6d.? Ans. 1lb. 4oz. $3\frac{479}{627}$pwts.

7. If the carriage of 13 cwt. 1 qr. for 72 miles be £2. 10s. 6d., what will be the carriage of 7 cwt. 3 qrs. for 112 miles?
Ans. £2 5s. 11d. $1\frac{77}{159}$ q.

8. A wall, to be built to the height of 27 feet, was raised to the height of 9 ft. by 12 men in 6 days; how many men must be employed to finish the wall in 4 days at the same rate of working? Ans. 36 men.

9. If a regiment of soldiers, consisting of 939 men, can eat 351 quarters of wheat in 7 months; how many soldiers will eat 1464 quarters in 5 months, at that rate? Ans. $5483\frac{23}{195}$.

10. If 248 men, in 5 days of 11 hours each, dig a trench 230 yards long, 3 wide and 2 deep; in how many days of 9 hours each, will 24 men dig a trench of 420 yards long, 5 wide and 3 deep? Ans. $288\frac{59}{367}$.

11. If 6 men build a wall 20 ft. long, 6 ft. high, and 4 ft. thick, in 16 days, in what time will 24 men build one 200 ft. long, 8 ft. high, and 6 ft. thick? Ans. 80 days.

12. If the freight of 9 hhds. of sugar, each weighing 12 cwt., 20 leagues, cost £16, what must be paid for the freight of 50 tierces, each weighing $2\frac{1}{2}$ cwt. 100 leagues? Ans. £92 11s. $10\frac{2}{9}$d.

13. If 4 reapers receive $11.04 for 3 days' work, how many men may be hired 16 days for $103.04? Ans. 7 men.

14. If 7 oz. 5 pwt. of bread be bought for $4\frac{3}{4}$d. when corn is 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price per bushel is 5s. 6d.? Ans. 1 lb. 4 oz. $3\frac{479}{627}$ pwts.

15. If $100 gain $6 in 1 year, what will 400 gain in 9 months?

16. If $100 gain $6 in one year, in what time will $400 gain $18?

17. If $400 gain $18 in 9 months, what is the rate per cent. per annum?

18. What principal, at 6 per cent. per ann., will gain $18 in 9 months?

19. A usurer put out $75 at interest, and, at the end of 8 months, received, for principal and interest, $79; I demand at what rate per cent. he received interest. Ans. 8 per ct.

20. If 3 men receive £$8\frac{9}{10}$ for $19\frac{1}{2}$ days' work, how much must 20 men receive for $100\frac{1}{4}$ days? Ans. £305 0s. 8d.

21. If 40 men in 10 days, can reap 200 acres of grain, how many acres can 14 men reap in 24 days? Ans. 168 acres.

22. If 14 men in 24 days, can reap 168 acres of grain; how many acres can 40 men reap in 10 days? Ans. 200 acres.

23. If 16 men in 32 days, can mow 256 acres of grass; in how many days will 8 men mow 96 acres? Ans. 24 days.

24. If 4 men mow 96 acres in 12 days; how many acres can 8 men mow in 16 days? Ans. 256.

25. If a family of 16 persons spend $320 in 8 months; how much would 8 of the same family spend in 24 months? Ans. $480.

26. If a family of 8 persons in 24 months spend $480; how much would they spend if their number were doubled, in 8 months? Ans. $320.

27. If 12 men build a wall 100 ft. long, 4 ft. high, and 3 ft. thick, in 40 days; in what time will 6 men build one, 20 ft. long, 6 ft. high, and 4 ft. thick?

FELLOWSHIP.

The *Rule of Fellowship*, is a method of ascertaining the respective gains or losses of individuals engaged in joint trade.

Let the pupils perform the following sums as a mental exercise.

1. Two men own a ticket; the first owns $\frac{1}{4}$, and the second owns $\frac{3}{4}$ of it; the ticket draws a prize of 40 dollars; what is each man's share of the money?

2. Two men purchase a ticket for 4 dollars, of which one man pays 1 dollar, and the other 3 dollars; the ticket draws 40 dollars; what is each man's share of the money?

3. A and B bought a quantity of cotton; A paid $100, and B $200; they sold it so as to gain $30; what were their respective shares of the gain?

The value of what is employed in trade is called the *Capital*, or *Stock*. The gain or loss to be shared is called the *Dividend*.

Each man's gain or loss is always in proportion to his share of the stock, and on this principle the rule is made.

RULE. *As the whole stock is to each man's share of the stock, so is the whole gain or loss, to his share of the gain or loss.*

4. Two persons have a joint stock in trade; A put in $250, and B $350; they gain $400; what is each man's share of the profit?

What is Fellowship? What is stock? Dividend? What is the rule?

OPERATION.

A's stock, \$250 } Then,
B's stock, \$350 } 600 : 250 : : 400 : \$166.666$\frac{2}{3}$ A's gain.
Whole stock \$600 } 600 : 350 : : 400 : \$233.333$\frac{1}{3}$ B's gain.

The pupil will perceive that the process may be contracted by cutting off an equal number of ciphers from the *first* and *second*, or *first* and *third* terms; thus, 6 : 250 : 4 : 166.666$\frac{2}{3}$, &c.

It is obvious the correctness of the work may be ascertained by finding whether the sums of the *shares* of the gains are equal to the *whole* gain; thus, \$166.666$\frac{2}{3}$+\$233.333$\frac{1}{3}$=\$400, whole gain.

5. A, B, and C, trade in company: A's capital was \$175, B's \$200, and C's \$500; by misfortune they lose \$250; what loss must each sustain?

Ans. { \$ 50., A's loss.
\$ 57.142$\frac{6}{7}$, B's loss.
\$142.857$\frac{1}{7}$, C's loss. }

6. Divide 600 among 3 persons, so that their shares may be to each other as 1, 2, 3, respectively. Ans. \$100, \$200, and \$300.

In assessing taxes, it is customary to obtain an inventory of every man's property, in the whole town, and also a list of the number of polls. Each poll is rated at a tax of a certain value. From the *whole* tax to be raised, is taken out what the tax on polls amounts to, and the remainder of the tax is to be assessed on the property in the town.

We may then find the tax upon 1 dollar, and make a table containing the taxes on 1, 2, 3, &c. to 10 dollars; then on 20, 30, &c. to 100 dollars; and then on 100, 200, &c. to 1000 dollars. Then, knowing the inventory of any individual, it is easy to find the tax upon his property.

TABLE.

	dolls.		cts.		dolls.		cts.		dolls.		dolls.
Tax on	1	is	,03	Tax on	10	is	,30	Tax on	100	is	3,
"	2	"	,06	"	20	"	,60	"	200	"	6,
"	3	"	,09	"	30	"	,90	"	300	"	9,
"	4	"	,12	"	40	"	1,20	"	400	"	12,
"	5	"	,15	"	50	"	1,50	"	500	"	15,
"	6	"	,18	"	60	"	1,80	"	600	"	18,
"	7	"	,21	"	70	"	2,10	"	700	"	21,
"	8	"	,24	"	80	"	2,40	"	800	"	24,
"	9	"	,27	"	90	"	2,70	"	900	"	27,
								"	1000	"	30,

1. A certain town, valued at $ 64530, raises a tax of $ 2259,90; there are 540 polls, which are taxed $,60 each; what is the tax on a dollar, and what will be A's tax, whose *real estate* is valued at $ 1340, his personal property at $ 874, and who pays for 2 polls?

540 × ,60=$ 324, amount of the poll taxes, and $ 2259,90,—$ 324=1935,90, to be assessed on property. $ 64530 : $ 1935,90 : : $ 1 : ,03 or, $\frac{1935,90}{64530}$=,03, tax on $ 1.

Now, to find A's tax, his real estate being $ 1340, I find by the table, that

The tax on	$ 1000	-	is	-	30,
The tax on	300	-	-	-	9,
The tax on	40	-		-	1,20
Tax on his real estate		-		-	$ 40,20
In like manner I find the tax on his personal property to be					26,22
2 polls, at ,60 each, are					1,20
				Amount,	$ 67,62

2. What will B's tax amount to, whose inventory is 874 dollars *real*, and 210 dollars *personal* property, and who pays for 3 polls? Ans. $ 34,32.

3. What will be the tax of a man paying for 1 poll, whose property is valued at $ 3482?——at $ 768?——

Ans. to the last, $ 140.31.

Two men paid 10 dollars for the use of a pasture 1 month; A kept in 24 cows, and B 16 cows; how much should each pay?

4. Two men hired a pasture for $ 10; A put in 8 cows 3 months, and B put in 4 cows 4 months; how much should each pay?

The pasturage of 8 cows for 3 months is the same as of 24 cows for 1 month, and the pasturage of 4 cows for 4 months is the same as of 16 cows for 1 month. The shares of A and B, therefore, are 24 to 16, as in the former question. Hence, when *time* is regarded in fellowship,—*Multiply each one's stock by the time he continues in the trade, and use the product for his share.* This is called *Double Fellowship.*

Ans. A 6 dollars, and B 4 dollars.

5. A and B enter into partnership; A puts in $ 100 6 months,

What is the rule when time is regarded in Fellowship?

and then puts in $50 more; B puts in $200 4 months, and then takes out $80; at the close of the year they find that they have gained $95; what is the profit of each?

Ans. { $43,711, A's share.
{ $51,288, B's share.

6. A, with a capital of $500, began trade, Jan. 1, 1826, and meeting with success, took in B as a partner, with a capital of 600, on the first of March following; four months after, they admit C as a partner, who brought $800 stock; at the close of the year they find the gain to be $700; how must it be divided among the partners?

Ans. { $250, A's share.
{ $250, B's share.
{ $200, C's share.

ALLIGATION.

The rule of Alligation teaches how to gain the *mean value* of a mixture that is made by uniting several articles of *different values.*

Alligation Medial, teaches how to obtain the value, (or *mean price*,) of a mixture, when the *quantities* and *prices* of the several articles are given.

RULE. *As the whole mixture is to the whole value, so is any part of the composition, to its mean price.*

EXAMPLES.

1. A farmer mixed 15 bushels of rye, at 64 cents a bushel, 18 bushels of Indian corn, at 55 cts. a bushel, and 21 bushels of oats, at 28 cts. a bushel; I demand what a bushel of this mixture is worth?

bu.	*cts.*	*$ cts.*
15 at	64 =	9,60
18	55 =	9.90
21	28 =	5,88
54		25,38

bu. *$ cts.* *bu.*
As 54 : 25,38 : : 1
1
——— *cts.*
54)25,38(.47 Ans.

2. If 20 bushels of wheat at 1 dol. 35 cts. per bushel, be mixed with 10 bushels of rye at 90 cents per bushel, what will a bushel of this mixture be worth? Ans. $1,20 *cts.*

3. A tobacconist mixed 36 lbs. of tobacco, at 1s. 6d. per lb.,

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What is Alligation? What is Alligation medial? What is the rule?

12 lbs. at 2s. a pound, with 12 lbs. at 1s. 10d. per lb., what is the price of a pound of this mixture? Ans. 1s. 8d.

4. A grocer mixed 2 C. of sugar at 56s. per C. and 1 C. at 43s. per C. and 2 C. at 50s. per C. together; I demand the price of 3 cwt. of this mixture? Ans. £7. 13s.

5. A wine merchant mixes 15 gallons of wine at 4s. 2d. per gallon, with 24 gallons at 6s. 8d. and 20 gallons at 6s. 3d.; what is a gallon of this composition worth? Ans. 5s. 10d. $2\frac{42}{59}$ qrs.

Alligation Alternate, teaches how to find the *quantity* of each article, when the *mean price* of the whole mixture, and also the *prices* of *each separate article* are known.

RULE. *Reduce the mean price and the prices of each separate article to the same order.*

Connect with a line each price that is less than the mean price, with one or more that is greater; and each price greater than the mean price, with one or more that is less.

Write the difference between the mean price, and the price of each separate article, opposite the price with which it is connected; then the sum of the differences, standing against any price, will express the relative quantity to be taken of that price.

EXAMPLES.

1. A merchant has several kinds of tea; some at 8 shillings, some at 9 shillings, some at 11 shillings, and some at 12 shillings per pound; what proportions of each must he mix, that he may sell the compound at 10s. per pound?

The pupil will perceive, that there may be as many different ways of mixing the simples, and consequently as many different answers, as there are different ways of linking the several prices.

OPERATIONS.

```
      ( 8s.  -2 )
10s. {  9s.  -1  } Ans.
      ( 11s. -1 )
      ( 12s. -2 )

Or,  ( 8   -2+1=3 )
10  {  9    1   =1 } Ans.
     ( 11   1+2=3 )
     ( 12  -2   =2 )
```

Here the prices of the simples are set one directly under another, in order, from least to greatest, and the mean rate, (10s.) written at the left hand. In the first way of linking, we take in the proportion of 2 pounds of the teas at 8 and 12s. to 1

What is Alligation alternate? What is the rule?

pound at 9 and 11s. In the second way, we find for the answer, 3 pounds at 8 and 11s. to 1 pound at 9 and 12s.

2. What proportions of sugar, at 8 cents, 10 cents, and 14 cents per pound, will compose a mixture worth 12 cents per pound?

Ans. In the proportion of 2 lbs. at 8 and 10 cts., to 6 lbs. at 14 cents.

NOTE. As these quantities only express the *proportions* of each kind, it is plain, that a compound of the *same mean price* will be formed by taking 3 times, 4 times, one half, or any proportion, of each quantity. Hence,

When the quantity of one simple is given, after finding the *proportional* quantities, by the above rule, we may say, *As the* PROPORTIONAL *quantity: is to the* GIVEN *quantity : : so is each of the other* PROPORTIONAL *quantities : to the* REQUIRED *quantities of each.*

3. If a man wishes to mix 1 gallon of brandy worth 16s. with rum at 9s. per gallon, so that the mixture may be worth 11s. per gallon, how much rum must he use?

Taking the differences as above, we find the *proportions* to be 2 of brandy to 5 of rum; consequently, 1 gallon of brandy will require 2½ gallons of rum. Ans. 2½ gallons.

4. A grocer has sugars worth 7 cents, 9 cents, and 12 cents per pound, which he would mix so as to form a compound, worth 10 cents per pound; what must be the *proportions* of each kind?

Ans. 2 lbs. of the first and second, to 4 lbs. of the 3d. kind.

5. If he use 1 lb. of the first kind, how much must he take of the others? —— if 4 lbs., what? —— if 6 lbs., what? —— if 10 lbs., what? —— if 20 lbs., what?

Ans. to the last, 20 lbs. of the 2d, and 40 of the 3d.

6. A merchant has spices at 16d. 20d. and 32d. per pound; he would mix 5 pounds of the first sort with the others, so as to form a compound worth 24d. per pound; how much of each sort must he use?

Ans. 5 lbs. of the second, and 7½ lbs. of the third.

7. How many gallons of water, of no value, must be mixed with 60 gallons of rum, worth 80 cents per gallon, to reduce its value to 70 cents per gallon? Ans. 8$\frac{4}{7}$ galls.

8. A man would mix 4 bushels of wheat, at $1,50 per bushel,

How can the required quantities of each of the simples be obtained?

rye at $1,16, corn at 75 c. and barley at 50 c. so as to sell the mixture at 84 c. per bushel; how much of each may he use?

When the *quantity* of the compound is given, we may say, *As the sum of the* PROPORTIONAL *quantities, found by the above rule, is to the quantity* REQUIRED, *so is each* PROPORTIONAL *quantity, found by the rule, to the* REQUIRED *quantity of* EACH.

9. A man would mix 100 pounds of sugar, some at 8 cents, some at 10 cents, and some at 14 cents per pound, so that the compound may be worth 12 cents per pound; how much of each kind must he use?

We find the proportions to be, 2, 2, and 6. Then, 2+2+6 =10, and

$$10 : 100 :: \left\{\begin{array}{l} 2 : 20 \text{ lbs. at } 8 \text{ cts.} \\ 2 : 20 \text{ lbs. at } 10 \text{ cts.} \\ 6 : 60 \text{ lbs. at } 14 \text{ cts.} \end{array}\right\} \text{Ans.}$$

10. How many gallons of water, of no value, must be mixed with brandy at $1,20 per gallon, so as to fill a vessel of 75 gallons, which may be worth 92 cents per gal.?

Ans. 17½ gallons of water to 57½ gallons of brandy.

11. A grocer has currants at 4d., 6d., 9d., and 11d. per lb.; and he would make a mixture of 240 lbs., so that the mixture may be sold at 8d. per lb.; how many pounds of each sort may he take? Ans. 72, 24, 48, and 96 lbs., or 48, 48, 72, 72, &c.

NOTE. This question may have five different answers.

DUODECIMALS.

Duodecimal is derived from the Latin word *duodesim*, signifying *twelve*.

They are fractions of *a foot*, which is supposed to be divided into *twelve* equal parts called *primes*, marked thus, (′). Each prime is supposed to be subdivided into 12 equal parts called *seconds*, marked thus (″). Each second is also supposed to be divided into twelve equal parts called *thirds*, marked thus (‴), and so on to any extent.

It thus appears that

1′ an inch or prime is $\frac{1}{12}$ of a foot.

1″ a second is $\frac{1}{12}$ of $\frac{1}{12}$ or $\frac{1}{144}$ of a foot.

1‴ a third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, or $\frac{1}{1728}$ of a foot, &c.

Whenever therefore any number of *seconds*, (as 5″) are men-

What are duodecimals?

tioned, it is to be understood as so many $\frac{1}{144}$ of a foot, and so of the *thirds, fourths, &c.*

Duodecimals are added and subtracted like other compound numbers, 12 of a less order making 1 of the next higher, thus,

12'''' fourths make 1 third 1'''.

12''' thirds make 1 second 1''.

12'' seconds make 1 prime or inch 1'.

12' inches or primes, make 1 foot.

The addition and subtraction of Duodecimals is the same as other compound numbers.

These marks ' '' ''' '''' are called *indices.*

MULTIPLICATION OF DUODECIMALS.

Duodecimals are chiefly used in measuring *surfaces* and *solids.*

How many square feet in a board 16 feet 7 inches long, and 1 foot 3 inches wide?

NOTE. The square contents of any thing are found by multiplying the length into the breadth.

The following example is explained above.

EXAMPLES.

16	7'	
1	3'	
16	7'	
4	1'	9''
20	8'	9''

It is generally more convenient to multiply by the *higher orders* of the multiplier *first.*

Thus we begin and multiply the multiplicand first by the 1 foot, and set the answers down as above.

We then multiply by the 3' or $\frac{3}{12}$ of a foot. 16 is changed to a fraction, thus $\frac{16}{1}$, and this multiplied by $\frac{3}{12}$ is $\frac{48}{12}$, or 48', which is 4 feet, (for there are 12' in every foot,) and is set under that order.

We now multiply 7' (or $\frac{7}{12}$) by 3' (or $\frac{3}{12}$) and the answer is $\frac{21}{144}$ or 21''.

This is 1' to set under the order of *twelfths*, and 9'' $\left(\frac{9}{144}\right)$ to be set under the order of *seconds.*

The two products are then added together, and the answer is obtained, which is 20 feet 8 primes 9 seconds.

Another example will be given in which the *cubic* contents of a block are found by multiplying the *length, breadth* and *thickness* together.

How many solid feet in a block 15 ft. 8' *long*, 1 ft. 5' *wide*, and 1 ft. 4' *thick?*

When are duodecimals used?

OPERATION.

Let this example be studied and understood before the rule is learned. If any difficulty is found, let both multiplier and multiplicand be expressed as *Vulgar Fractions*, and then multiply.

In duodecimals it is always the case that the *product* of two orders, will belong to that order which is made by *adding the indices of the factors.*

Length,	15	8′		
Breadth,	1	5′		
	15	8′		
	6	6′	4″	
	22	2′	4″	
Thickness	1	4′		
	22	2′	4″	
	7	4′	9″	4‴
Ans.	29	7′	1″	4‴

RULE. *Write the figures as in the addition of compound numbers. Multiply by the higher orders of the multiplier first, remembering that the product of two orders belongs to the order denoted by the sum of their indices.*

If any product is large enough to contain units of a higher order, change them to a higher order, and place them where they belong.

EXAMPLES.

How many square feet in a pile of boards 12 ft. 8′ long, and 13′ wide?

What is the product of 371 ft. 2′ 6″ multiplied by 181 ft. 1′ 9″?
Ans. 67242 ft. 10′ 1″ 4‴ 6⁗.

If a floor be 10 ft. 4′ 5″ long, and 7 ft. 8′ 6″ wide, what is its surface? Ans. 79 ft. 11′ 0″ 6‴ 6⁗.

What is the solidity of a wall 53 ft. 6′ long, 10 ft. 3′ high, and 2 ft. thick? Ans. 1096¾ ft.

INVOLUTION.

When a number is multiplied into itself, it is said to be *involved*, and the process is called *Involution.*

Thus 2×2×2 is 8. Here the number 2 is multiplied into itself twice.

The *product* which is obtained by multiplying a number into itself, is called a *Power.*

Thus, when 2 is multiplied into itself *once*, it is 4, and this is

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What is the rule? What is involution? What is a power? root?

called the *second power* of 2. If it is multiplied into itself *twice* ($2\times2\times2=8$) the answer is 8, and this is called the *third power*.

The number which is involved, is called the *Root*, or *first power*.

Thus, 2 is the root of its *second* power 4, and the root of its *third* power 8.

A power is named, or numbered, according to the number of times its root is used as a factor. Thus the number 4 is called the *second* power of its root 2, because the root is *twice* used as a factor; thus, $2\times2=4$.

The number 8 is called the *third power* of its root 2; because the root is used *three times* as a factor; thus, $2\times2\times2=8$.

The method of expressing a power, is by writing its root, and then above it placing a small figure, to show the number of times that *the root is used as a factor*.

Thus the *second power* of 2 is 4, but instead of writing the product 4, we write it thus, 2^2.

The third power of 2 is written thus, 2^3.

The fourth power of 2 is 16, and is written thus, 2^4.

The small figure that indicates *the number of times that the root is used as a factor*, is called the *Index*, or *Exponent*.

The different powers have other names beside their *numbers*.

Thus, the *second power* is called the *Square*.

The *third power* is called the *Cube*.

The *fourth power* is called the *Biquadrate*.

The *fifth power* is called the *Sursolid*.

The *sixth power* is called the *Square-cubed*.

Powers are *indicated* by exponents. When a power is actually found by multiplication, *involution* is said to be *performed*, and the *number* or *root* is *involved*.

RULE OF INVOLUTION.

To involve a number, multiply it into itself, as often as there are units in the exponent, save once.

NOTE.—The reason why it is multiplied *once less* than there are units in the exponent, is, that the first time the number is multiplied, the root is used *twice* as a factor; and the exponent shows, not how many times we are to multiply, but *how many times the root is used as a factor*.

1. What is the cube of 5? Ans. $5\times5\times5=125$

2. What is the fourth power of 4? Ans. 256.

How is a power named? What are the names of the different powers? What is the rule for involution?

3. What is the square of 14? Ans. 196.
4. What is the cube of 6? Ans. 216.
5. What is the 5th power of 2? Ans. 32.
6. What is the 7th power of 2? Ans. 128.
7. What is the square of $\frac{1}{2}$? Ans. $\frac{1}{4}$.
8. What is the cube of $\frac{2}{3}$? Ans. $\frac{8}{27}$.

A Fraction is involved, by involving both numerator and denominator.

9. What is the fourth power of $\frac{3}{5}$? Ans. $\frac{81}{625}$.
10. What is the square of $5\frac{1}{2}$? Ans. $30\frac{1}{4}$.
11. What is the square of $30\frac{1}{4}$? Ans. $915\frac{1}{16}$.
12. Perform the involution of 8^5. Ans. 32,768.
13. Involve $\frac{4}{39}$, $\frac{11}{12}$, and $\frac{8}{9}$ to the third power each. Ans. $\frac{64}{59319}$; $\frac{1331}{1728}$; $\frac{512}{729}$.
14. Involve 211^3. Ans. 9,393,931.
15. Raise 25 to the fourth power. Ans. 390,625.
16. Find the sixth power of 1.2. Ans. 2.985,984.

EVOLUTION.

Evolution is the process of *finding the root of any number;* that is, of finding that number which, multiplied into itself, will produce the given number.

The *Square Root*, or *Second Root*, is a number which being *squared* (i. e. multiplied once into itself) will produce the given number. It is expressed either by this sign, put before a number, thus $\sqrt{4}$, or by the fraction $\frac{1}{2}$ placed above a number, thus, $4^{\frac{1}{2}}$.

The *Cube Root*, or *Third Root*, is a number, which being *cubed*, or multiplied by itself *twice*, will produce the given number. It is expressed thus, $\sqrt[3]{12}$; or thus, $12^{\frac{1}{3}}$.

All the other roots are expressed in the same manner. Thus the *fourth root* has this sign $\sqrt[4]{\ }$ put before a number, or else $\frac{1}{4}$ placed above it.

The *sixth root* has $\sqrt[6]{\ }$ before it, or $\frac{1}{6}$ above it, &c.

There are some numbers whose roots cannot be precisely

How is a fraction involved? What is evolution? How are roots ex-[pr]essed?

obtained; but by means of *decimals*, we can *approximate* to the number which is the root.

Numbers whose roots can be exactly obtained, are called *rational numbers*.

Numbers whose precise roots cannot be obtained, are called *surd numbers*.

When the root of several numbers united by the sign + or — is indicated, a *vinculum*, or line is drawn from the sign of the root over the numbers. Thus, the square root of 36—8 is written $\sqrt{36-8}$.

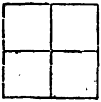
The root of a rational number, is a *rational root*, and the root of a surd number, is a *surd root*.

It is very necessary for practical purposes, to be able to find the amount of *surface* there is in any given quantity. For instance, if a man has 250 yards of matting, which is 2 yards wide, how much surface will it cover?

The rule for finding the *amount of surface*, is to *multiply the length by the breadth*, and this will give the amount of square *inches*, *feet*, or *yards*.

It is important for the pupil to learn the distinction between a *square quantity*, and a certain extent that is in the form of a *square*. For example, *four square inches*, and *four inches square* are different quantities.

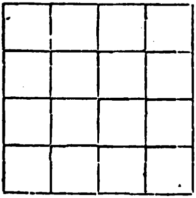
A



Four square inches may be represented in Fig. A. In this figure there are four *square inches*, but it makes a square which is only *two inches* on each side, or a *two inch square*.

A four inch square may be represented by Fig. B.

B.



Here the sides of the square are four inches long, and it is called *a four inch square*. But it contains *sixteen square inches*. For when the four inch square is cut into pieces of each an inch square, it will make sixteen of them.

A four inch square then, is a square whose *sides* are four inches long.

Four square inches are four squares that are each an inch on every side.

What are rational numbers? What are surd numbers? What is a rational root? Surd root? What is the rule for finding the amount of surface? What is the difference between an inch square, and a square inch?

When we wish to find the *square contents* of any quantity, we seek to know how many square *inches*, or *feet*, or *yards*, there are in the quantity given, and this is always found by multiplying the length by the breadth.

When the length and breadth of any quantity are given, we find its *square contents*, or the amount of surface it will cover, by multiplying the *length* by the *breadth*.

What are the square contents of 223 yds. of carpeting ⅞ wide?

What are the sq. contents of 249 yds. of matting ¾ wide?

If any quantity is placed in a *square form*, the *length of one side* is the *square root* of the square contents of this figure. Thus in the preceding example, B, the square contents of the figure are 16 square inches. The *side* of the square is 4 inches long; and 4 is the *square root* of 16. The square root, therefore, is the length of the sides of a square, made by the given quantity.

If we have *one side* of a square given, by the process of *Involution*, we find what are the *square contents* of the quantity given.

If, on the contrary, we have the *square contents* given, by the process of *Evolution*, we find what is the *length of one side* of the square, which can be made by the quantity given.

Thus if we have a *square* whose side is four inches, by *Involution* we find the surface, or *square contents* to be 16 *square inches*.

But if we have 16 *square inches* given, by *Evolution* we find what is *the length of one side* of the square made by these 16 inches.

EXTRACTION OF THE SQUARE ROOT.

Extracting the square root is finding a number, which, multiplied into itself, will produce the given number; or, it is finding the length of *one side* of a certain quantity, when that quantity is placed in an exact square.

It will be found by trial, that the root always contains just *half* as many, or one figure *more* than half as many figures as are in the given quantity. To ascertain, therefore, the number of figures in the required root, we point off the given number into periods of two figures each, beginning at the right, and

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-hat the explanation of the rule for extracting the square root?

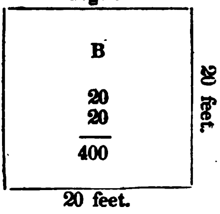
there will always be as many figures in the root as there are periods.

1. What is one side of a square, containing 784 square feet?

784(2
4
384

Pointing off as above, we find that the root will consist of *two* figures, a *ten* and a *unit*.

Fig. 1.



We now take the highest period 7 (hundreds), and ascertain how many feet there will be in the largest square that can be made of this quantity, the sides of which must be of the order of *tens*. No square larger than 4 (hundreds) can be obtained in 7 (hundreds), the sides of which will be each 20 feet (because $20\times20=400$). These 20 feet (or 2 tens) being sides of the square, are placed in the quotient as the first figure of the root.

This square may be represented by Fig. 1.

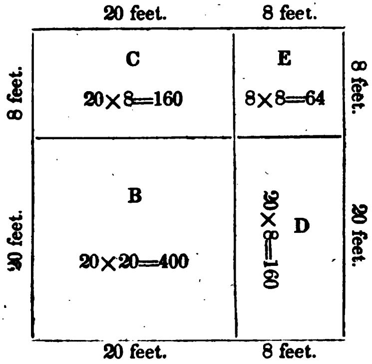
We now take out the 400 from 700, and 300 square feet remain. These are added to the next period (84 feet), making 384, which are to be arranged around the square B, in such a way as not to destroy its *square form;* consequently the additions must be made on *two* sides.

To ascertain the *breadth* of these additions, the 384 must be divided by the length of the two sides (20+20), and as the root already found is *one* side, we double this root for a divisor, making 4 tens or 40, for as 40 feet is the length of these sides, there will be as many feet in breadth as there are forties in 384. The quotient arising from the division is 8, which is the *breadth* of the addition to be made, and which is placed in the quotient, after the 4 tens;

784(28 Root.
4
48 | 384
384
000

But it will be seen by Fig. 2, that to complete the square, the corner E must be filled by a small square, the sides of which are each equal to the *width* of C and D, that is, 8 feet. Adding this to the 4 tens, or 40, we find that the whole *length* of the addition to be made around the square B, is 48 feet, instead of 40. This multiplied by its *breadth*, 8 feet (the quotient figure), gives the *contents* of the whole addition, viz. 384 feet.

Fig. 2.



As there is no remainder, the work is done, and 28 feet is the side of the given square.

The *proof* may be seen by involution, thus; 28×28=784; or it may be proved, by adding together the several parts of the figure, thus;

B	contains	400	feet.
C	"	160	"
D	"	160	"
E	"	64	"
	Proof	784	

If, in any case, there is a *remainder*, after the last period is brought down, it may be reduced to a decimal fraction, by annexing two ciphers for a new period, and the same process continued.

Whenever any dividend is too small to contain the divisor, a cipher must be placed in the root, and another period brought down.

From the above illustrations, we see the reasons for the following rule.

RULE FOR EXTRACTING THE SQUARE ROOT.

1. *Point off the given number, into periods of two figures each, beginning at the right.*

2. *Find the greatest square in the first left hand period, and subtract it from that period. Place the root of this square in the quotient. To the remainder bring down the next period for a dividend.*

3. *Double the root already found (understanding a cipher at the right) for a divisor. Divide the dividend by it, and place the quotient figure in the root, and also in the divisor.*

4. *Multiply the divisor, thus increased, by the last figure of the root, and subtract the product from the dividend. To the remainder bring down the next period, for a new dividend. Double the root already found, for a new divisor, and proceed as before.*

EXAMPLES.

What is the square root of 998001?

```
        998001(999 Root.
        81
    189)1880
        1701
   1989)17901
        000
```

Find the sq. root of 784. A. 28. Of 676. A. 26. Of 625. A. 25. Of 487,204. A. 698. Of 638,401. A. 779. Of 556,516. A. 746. Of 441. A. 21. Of 1024. A. 32. Of 1444. A. 38. Of 2916. A. 54. Of 6241. A. 79. Of 9801. A. 99. Of 17,956. A. 134. Of 32,761. A. 181. Of 39,601. A. 199. Of 488,601. A. 699.

Find the sq root of 69. A. 8.3066239. Of 83. A. 9.1104336. Of 97. A. 9.8488578. Of 299. A. 17.2916165. Of 222. A. 14.8996644. Of 282. A. 16.7928556. Of 394. A. 19.8494332. Of 351. A. 18.7349940. Of 699. A. 26.4386081. Of 979. A. 31.2889757. Of 989. A. 31.4483704. Of 999. A. 31.6069613. Of 397. A. 19.9248588. Of 687. A. 26.2106848. Of 892. A. 29.8663690.

It was shown in the article on Involution, that a fraction is involved by involving both numerator and denominator, hence to find the *root* of a fraction, *extract the root both of numerator and denominator*. If this cannot be done, the fraction may be reduced to a decimal, and its root extracted.

What is the square root of $\frac{25}{36}$? A. $\frac{5}{6}$. Of $\frac{160801}{249001}$? A. $\frac{401}{499}$. Of $\frac{237169}{480249}$? A. $\frac{487}{693}$. Of $\frac{430336}{483025}$? A. $\frac{656}{695}$. Of $\frac{616225}{617796}$? A. $\frac{785}{786}$. Of $\frac{606841}{942841}$? A. $\frac{779}{971}$.

What is the rule for extracting the square root?

Find the sq. root of $\frac{3}{4}$. A. .8660254. Of $\frac{5}{12}$. A. .645497. Of $17\frac{3}{8}$. A. 4.168333. Of $\frac{3}{80}$. A. .193649167. Of $\frac{9}{13}$. A. .83205. Of $\frac{5}{60}$ A. .288617394+.

EXTRACTION OF THE CUBE ROOT.

A *Cube* is a solid body, having six equal sides, each of which is an exact square. Thus a solid, which is 1 foot long, 1 foot high, and 1 foot wide, is a *cubic foot;* and a solid whose length, breadth, and thickness are each 1 yard, is called a *cubic yard.*

The root of a cube is always the *length* of one of its sides; for as the length, breadth, and thickness of such a body are the same, the length of one side, raised to the third power, will show the contents of the whole.

Extracting the Cube Root of any quantity, therefore, is finding a number, which multiplied into itself, *twice*, will produce that quantity;—or it is finding the length of one side of a given quantity, when that quantity is placed in an exact cube.

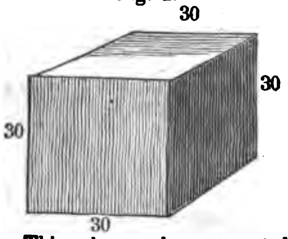
To ascertain the number of figures in a cube root, we point off the given number, into periods of three figures each, beginning at the right, and there will be as many figures in the required root as there are periods.

1. What is the length of one side of a cube, containing 32768 solid feet?

Pointing off as above, we find there will be *two* figures in the root, a *ten* and a *unit.*

$$\begin{array}{l} 32\dot{7}6\dot{8}(3 \\ 27 \\ \hline 5768 \end{array}$$

Fig. 1.



We now take the highest period, 32 (thousands), and ascertain what is the largest cube that can be contained in this quantity, the sides of which will be of the order of *tens*. No cube larger than 27 (thousands) can be contained in 32 (thousands). The sides of this are 3 tens or 30 (because $30 \times 30 \times 30 = 27{,}000$) which are placed as the first figure of the root.

This cube may be represented by Fig. 1.

We now take the 27000, from 32000, and 5000 solid feet remain. These are added to the next period (768), making 5768, which are to be arranged around the cubic figure 1, in such a way as not to destroy its cubic form; consequently the addition must be made to *three* of its sides.

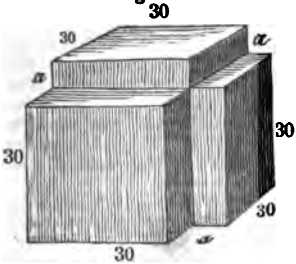
We must now ascertain, what will be the *thickness* of the addition made to each side. This will of course depend upon the *surface to be covered*. Now the length of one side has been shown to be 30 feet, and, as in a cube, the length and breadth of the sides are equal, multiplying the length of one side into itself will show the surface of *one* side, and this multiplied by 3, the *number of sides*, gives the contents of the surface of the *three* sides. Thus 30×30=900, which multiplied by 3=2700 feet.

```
   32768(32
   27
2700)5768
     5400
      360
        8
     5768
     0000
```

Now as we have 5768 solid feet to be distributed upon a surface of 2700 feet, there will be as many feet in the thickness of the addition, as there are twenty-seven hundreds in 5768. 2700 is contained in 5768 *twice;* therefore 2 feet is the thickness of the addition made to each of the three sides.

By multiplying this thickness, by the extent of surface (2700 × 2) we find that there are 5400 solid feet contained in these additions.

Fig. 2.



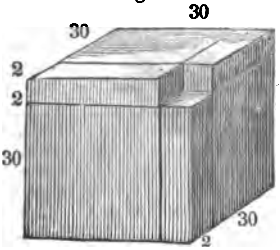
But if we examine Fig. 2, we shall find that these additions do not complete the cube, for the three corners *a a a* need to be filled by blocks of the same length as the sides (30 feet) and of the same breadth and thickness as the previous additions (viz. 2 feet.)

Now to find the *solid contents* of these blocks, or the number of feet required to fill these corners, we multiply the length, breadth, and thickness of *one* block together, and then multiply this product by 3, the *number* of blocks. Thus, the breadth and thickness of each block has been shown to be

2 feet; 2×2=4, and this multiplied by 30 (the *length*) =120, which is the solid contents of *one* block. But in *three*, there will be three times as many solid feet, or 360, which is the number required to fill the deficiencies.

In other words, we square the last quotient figure (2) multiply the product by the first figure of the quotient (3 tens) and then multiply the last product by 3, the number of deficiencies.

Fig. 3.

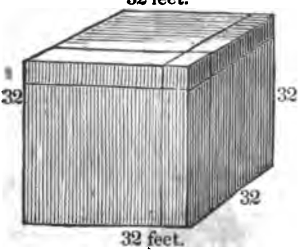


But by examining Fig. 3, it appears that the figure is not yet complete, but that a small cube is still wanting, where the blocks last added meet. The sides of this small cube, it will be seen, are each equal to the width of these blocks, that is, 2 feet. If each side is 2 feet long, the whole cube must contain 8 solid feet (because 2×2×2=8), and it will be seen by Fig. 4, that this just fills the vacant corner, and completes the cube.

We have thus found, that the additions to be made around the large cube (Fig. 1) are as follows.

5400 solid feet upon three sides, (Fig. 2).
360 " " to fill the corners *a a a*.
8 " " to fill the deficiency in Fig. 3.

Fig. 4.



Now if these be added together, their sum will be 5768 solid feet, which subtracted from the dividend leave no remainder and the work is done. 32 feet is therefore the length of one side of the given cube.

The *proof* may be seen by involving the side now found to the third power, thus; 32×32×32=32768; or it may be proved by adding together the contents of the several parts, thus,

27000 feet = contents of Fig. 1.
5400 " = addition to the three sides.
360 " = addition to fill the corners *a a a*.
8 " = addition to fill the corner in Fig. 3.

32768 Proof.

From these illustrations we see the reasons for the following rule.

RULE FOR EXTRACTING THE CUBE ROOT.

1. *Point off the given number, into periods of three figures each, beginning at the right.*

2. *Find the greatest cube in the left hand period, and subtract it from that period. Place the root in the quotient, and to the remainder bring down the next period, for a dividend.*

3. *Square the root already found (understanding a cipher at the right) and multiply it by 3 for a divisor.*

Divide the dividend by the divisor, and place the quotient for the next figure of the root.

4. *Multiply the divisor by this quotient figure. Multiply the square of this quotient figure by the former figure or figures of the root, and this product by three. Finally cube this quotient figure, and add these three results together for a subtrahend.*

5. *Subtract the subtrahend from the dividend. To the remainder bring down the next period, for a new dividend, and proceed as before.*

If it happens in any case, that the divisor is not contained in the dividend, or if there is a remainder after the last period is brought down, the same directions may be observed, that were given respecting the ***square root***. (See page 218.)

EXAMPLES.

What is the cube root of 373248? 373248(72

343

$70^2 \times 3 =$ 14700)30248(First Dividend.

29400

$2^2 \times 70 \times 3 =$ 840

$2^3 =$ 8

30248 Subtrahend.

0000

Repeat the process of illustration. What is the rule for extracting the cube root?

3. What is the square of 14? Ans. 196.
4. What is the cube of 6? Ans. 216.
5. What is the 5th power of 2? Ans. 32.
6. What is the 7th power of 2? Ans. 128.
7. What is the square of $\frac{1}{2}$? Ans. $\frac{1}{4}$.
8. What is the cube of $\frac{2}{3}$? Ans. $\frac{8}{27}$.

A Fraction is involved, by involving both numerator and denominator.

9. What is the fourth power of $\frac{3}{5}$? Ans. $\frac{81}{625}$.
10. What is the square of $5\frac{1}{2}$? Ans. $30\frac{1}{4}$.
11. What is the square of $30\frac{1}{4}$? Ans. $915\frac{1}{16}$.
12. Perform the involution of 8^5. Ans. 32,768.
13. Involve $\frac{4}{39}$, $\frac{11}{12}$, and $\frac{8}{9}$ to the third power each. Ans. $\frac{64}{59319}$; $\frac{1331}{1728}$; $\frac{512}{729}$.
14. Involve 211^3. Ans. 9,393,931.
15. Raise 25 to the fourth power. Ans. 390,625.
16. Find the sixth power of 1.2. Ans. 2.985,984.

EVOLUTION.

Evolution is the process of *finding the root of any number;* that is, of finding that number which, multiplied into itself, will produce the given number.

The *Square Root*, or *Second Root*, is a number which being *squared* (i. e. multiplied once into itself) will produce the given number. It is expressed either by this sign, put before a number, thus $\sqrt{4}$, or by the fraction $\frac{1}{2}$ placed above a number, thus, $4^{\frac{1}{2}}$.

The *Cube Root*, or *Third Root*, is a number, which being *cubed*, or multiplied by itself *twice*, will produce the given number. It is expressed thus, $\sqrt[3]{12}$; or thus, $12^{\frac{1}{3}}$.

All the other roots are expressed in the same manner. Thus the *fourth root* has this sign $\sqrt[4]{}$ put before a number, or else $\frac{1}{4}$ placed above it.

The *sixth root* has $\sqrt[6]{}$ before it, or $\frac{1}{6}$ above it, &c.

There are some numbers whose roots cannot be precisely

How is a fraction involved? What is evolution? How are roots expressed?

obtained; but by means of *decimals*, we can *approximate* to the number which is the root.

Numbers whose roots can be exactly obtained, are called *rational numbers*.

Numbers whose precise roots cannot be obtained, are called *surd numbers*.

When the root of several numbers united by the sign + or — is indicated, a *vinculum*, or line is drawn from the sign of the root over the numbers. Thus, the square root of 36—8 is written $\sqrt{36-8}$.

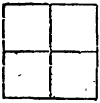
The root of a rational number, is a *rational root*, and the root of a surd number, is a *surd root*.

It is very necessary for practical purposes, to be able to find the amount of *surface* there is in any given quantity. For instance, if a man has 250 yards of matting, which is 2 yards wide, how much surface will it cover?

The rule for finding the *amount of surface*, is to *multiply the length by the breadth*, and this will give the amount of square *inches*, *feet*, or *yards*.

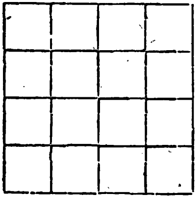
It is important for the pupil to learn the distinction between a *square quantity*, and a certain extent that is in the form of a *square*. For example, *four square inches*, and *four inches square* are different quantities.

A



Four square inches may be represented in Fig. A. In this figure there are four *square inches*, but it makes a square which is only *two inches* on each side, or a *two inch square*.

B.



A four inch square may be represented by Fig. B.

Here the sides of the square are four inches long, and it is called *a four inch square*. But it contains *sixteen square inches*. For when the four inch square is cut into pieces of each an inch square, it will make sixteen of them.

A four inch square then, is a square whose *sides* are four inches long.

Four square inches are four squares that are each an inch on every side.

What are rational numbers? What are surd numbers? What is a rational root? Surd root? What is the rule for finding the amount of surface? What is the difference between an inch square, and a square inch?

When we wish to find the *square contents* of any quantity, we seek to know how many square *inches*, or *feet*, or *yards*, there are in the quantity given, and this is always found by multiplying the length by the breadth.

When the length and breadth of any quantity are given, we find its *square contents*, or the amount of surface it will cover, by multiplying the *length* by the *breadth*.

What are the square contents of 223 yds. of carpeting $\frac{7}{8}$ wide?

What are the sq. contents of 249 yds. of matting $\frac{3}{4}$ wide?

If any quantity is placed in a *square form*, the *length of one side* is the *square root* of the square contents of this figure. Thus in the preceding example, B, the square contents of the figure are 16 square inches. The *side* of the square is 4 inches long; and 4 is the *square root* of 16. The square root, therefore, is the length of the sides of a square, made by the given quantity.

If we have *one side* of a square given, by the process of *Involution*, we find what are the *square contents* of the quantity given.

If, on the contrary, we have the *square contents* given, by the process of *Evolution*, we find what is the *length of one side* of the square, which can be made by the quantity given.

Thus if we have a *square* whose side is four inches, by *Involution* we find the surface, or *square contents* to be 16 *square inches*.

But if we have 16 *square inches* given, by *Evolution* we find what is *the length of one side* of the square made by these 16 inches.

EXTRACTION OF THE SQUARE ROOT.

Extracting the square root is finding a number, which, multiplied into itself, will produce the given number; or, it is finding the length of *one side* of a certain quantity, when that quantity is placed in an exact square.

It will be found by trial, that the root always contains just *half* as many, or one figure *more* than half as many figures as are in the given quantity. To ascertain, therefore, the number of figures in the required root, we point off the given number into periods of two figures each, beginning at the right, and

---

Repeat the explanation of the rule for extracting the square root?

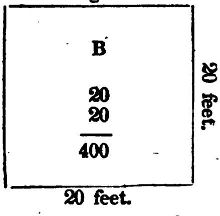
there will always be as many figures in the root as there are periods.

1. What is one side of a square, containing 784 square feet?

784(2
4
384

Pointing off as above, we find that the root will consist of *two* figures, a *ten* and a *unit*.

Fig. 1.



We now take the highest period 7 (hundreds), and ascertain how many feet there will be in the largest square that can be made of this quantity, the sides of which must be of the order of *tens*. No square larger than 4 (hundreds) can be obtained in 7 (hundreds), the sides of which will be each 20 feet (because $20\times20=400$). These 20 feet (or 2 tens) being sides of the square, are placed in the quotient as the first figure of the root.

This square may be represented by Fig. 1.

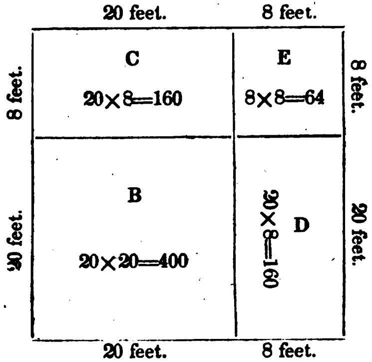
We now take out the 400 from 700, and 300 square feet remain. These are added to the next period (84 feet), making 384, which are to be arranged around the square B, in such a way as not to destroy its *square form*; consequently the additions must be made on *two* sides.

To ascertain the *breadth* of these additions, the 384 must be divided by the length of the two sides (20+20), and as the root already found is *one* side, we double this root for a divisor, making 4 tens or 40, for as 40 feet is the length of these sides, there will be as many feet in breadth as there are forties in 384. The quotient arising from the division is 8, which is the *breadth* of the addition to be made, and which is placed in the quotient, after the 4 tens;

784(28 Root.
4
48 | 384
384
000

But it will be seen by Fig. 2, that to complete the square, the corner E must be filled by a small square, the sides of which are each equal to the *width* of C and D, that is, 8 feet. Adding this to the 4 tens, or 40, we find that the whole *length* of the addition to be made around the square B, is 48 feet, instead of 40. This multiplied by its *breadth*, 8 feet (the quotient figure), gives the *contents of the* whole addition, viz. 384 feet.

Fig. 2.



As there is no remainder, the work is done, and 28 feet is the side of the given square.

The *proof* may be seen by involution, thus; 28×28=784; or it may be proved, by adding together the several parts of the figure, thus;

B	contains	400	feet.
C	"	160	"
D	"	160	"
E	"	64	"
	Proof	784	

If, in any case, there is a *remainder*, after the last period is brought down, it may be reduced to a decimal fraction, by annexing two ciphers for a new period, and the same process continued.

Whenever any dividend is too small to contain the divisor, a cipher must be placed in the root, and another period brought down.

From the above illustrations, we see the reasons for the following rule.

RULE FOR EXTRACTING THE SQUARE ROOT.

1. *Point off the given number, into periods of two figures each, beginning at the right.*

2. *Find the greatest square in the first left hand period, and subtract it from that period. Place the root of this square in the quotient. To the remainder bring down the next period for a dividend.*

3. *Double the root already found (understanding a cipher at the right) for a divisor. Divide the dividend by it, and place the quotient figure in the root, and also in the divisor.*

4. *Multiply the divisor, thus increased, by the last figure of the root, and subtract the product from the dividend. To the remainder bring down the next period, for a new dividend. Double the root already found, for a new divisor, and proceed as before.*

Examples.

What is the square root of 998001?

```
        998001(999 Root.
        81
    189)1880
        1701
   1989)17901
        000
```

Find the sq. root of 784. A. 28. Of 676. A. 26. Of 625. A. 25. Of 487,204. A. 698. Of 638,401. A. 779. Of 556,516. A. 746. Of 441. A. 21. Of 1024. A. 32. Of 1444. A. 38. Of 2916. A. 54. Of 6241. A. 79. Of 9801. A. 99. Of 17,956. A. 134. Of 32,761. A. 181. Of 39,601. A. 199. Of 488,601. A. 699.

Find the sq root of 69. A. 8.3066239. Of 83. A. 9.1104336. Of 97. A. 9.8488578. Of 299. A. 17.2916165. Of 222. A. 14.8996644. Of 282. A. 16.7928556. Of 394. A. 19.8494332. Of 351. A. 18.7349940. Of 699. A. 26.4386081. Of 979. A. 31.2889757. Of 989. A. 31.4483704. Of 999. A. 31.6069613. Of 397. A. 19.9248588. Of 687. A. 26.2106848. Of 892. A. 29.8663690.

It was shown in the article on Involution, that a fraction is involved by involving both numerator and denominator, hence to find the *root* of a fraction, *extract the root both of numerator and denominator*. If this cannot be done, the fraction may be reduced to a decimal, and its root extracted.

What is the square root of $\frac{25}{36}$? A. $\frac{5}{6}$. Of $\frac{160801}{249001}$? A. $\frac{401}{499}$. Of $\frac{237169}{480249}$? A. $\frac{487}{693}$. Of $\frac{430336}{483025}$? A. $\frac{656}{695}$. Of $\frac{616225}{617796}$? A. $\frac{785}{786}$. Of $\frac{606841}{942841}$? A. $\frac{779}{971}$.

What is the rule for extracting the square root?

Find the sq. root of $\frac{3}{4}$. A. .8660254. Of $\frac{5}{12}$. A. .645497. Of $17\frac{3}{8}$. A. 4.168333. Of $\frac{3}{80}$. A. .193649167. Of $\frac{9}{13}$. A. .83205. Of $\frac{5}{60}$ A. .288617394+.

EXTRACTION OF THE CUBE ROOT.

A *Cube* is a solid body, having six equal sides, each of which is an exact square. Thus a solid, which is 1 foot long, 1 foot high, and 1 foot wide, is a *cubic foot;* and a solid whose length, breadth, and thickness are each 1 yard, is called a *cubic yard.*

The root of a cube is always the *length* of one of its sides; for as the length, breadth, and thickness of such a body are the same, the length of one side, raised to the third power, will show the contents of the whole.

Extracting the Cube Root of any quantity, therefore, is finding a number, which multiplied into itself, *twice*, will produce that quantity;—or it is finding the length of one side of a given quantity, when that quantity is placed in an exact cube.

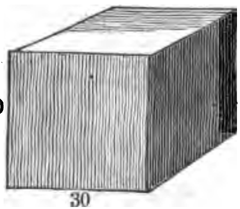
To ascertain the number of figures in a cube root, we point off the given number, into periods of three figures each, beginning at the right, and there will be as many figures in the required root as there are periods.

1. What is the length of one side of a cube, containing 32768 solid feet?

Pointing off as above, we find there will be *two* figures in the root, a *ten* and a *unit.*

```
32768(3
27
-----
 5768
```

Fig. 1.



We now take the highest period, 32 (thousands), and ascertain what is the largest cube that can be contained in this quantity, the sides of which will be of the order of *tens*. No cube larger than 27 (thousands) can be contained in 32 (thousands). The sides of this are 3 tens or 30 (because $30 \times 30 \times 30 = 27{,}000$) which are placed as the first figure of the root.

This cube may be represented by Fig. 1.

We now take the 27000, from 32000, and 5000 solid feet remain. These are added to the next period (768), making 5768, which are to be arranged around the cubic figure 1, in such a way as not to destroy its cubic form; consequently the addition must be made to *three* of its sides.

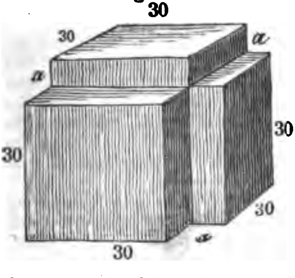
We must now ascertain, what will be the *thickness* of the addition made to each side. This will of course depend upon the *surface to be covered*. Now the length of one side has been shown to be 30 feet, and, as in a cube, the length and breadth of the sides are equal, multiplying the length of one side into itself will show the surface of *one* side, and this multiplied by 3, the *number of sides*, gives the contents of the surface of the *three* sides. Thus 30×30=900, which multiplied by 3=2700 feet.

```
   32768(32
   27
 ---------
 2700)5768
 ---------
      5400
       360
         8
      ----
      5768
      ----
      0000
```

Now as we have 5768 solid feet to be distributed upon a surface of 2700 feet, there will be as many feet in the thickness of the addition, as there are twenty-seven hundreds in 5768. 2700 is contained in 5768 *twice;* therefore 2 feet is the thickness of the addition made to each of the three sides.

By multiplying this thickness, by the extent of surface (2700 × 2) we find that there are 5400 solid feet contained in these additions.

Fig. 2.



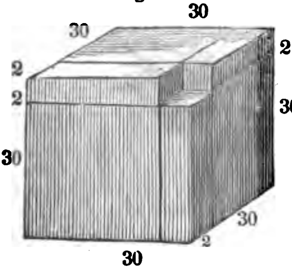
But if we examine Fig. 2, we shall find that these additions do not complete the cube, for the three corners *a a a* need to be filled by blocks of the same length as the sides (30 feet) and of the same breadth and thickness as the previous additions (viz. 2 feet.)

Now to find the *solid contents* of these blocks, or the number of feet required to fill these corners, we multiply the length, breadth, and thickness of *one* block together, and then multiply this product by 3, the *number* of blocks. Thus, the breadth and thickness of each block has been shown to be

2 feet; $2\times2=4$, and this multiplied by 30 (the *length*) $=120$, which is the solid contents of *one* block. But in *three*, there will be three times as many solid feet, or 360, which is the number required to fill the deficiencies.

In other words, we square the last quotient figure (2) multiply the product by the first figure of the quotient (3 tens) and then multiply the last product by 3, the number of deficiencies.

Fig. 3.

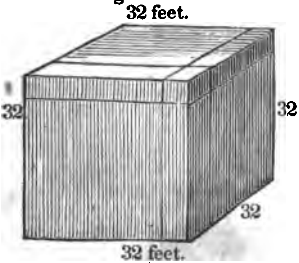


But by examining Fig. 3, it appears that the figure is not yet complete, but that a small cube is still wanting, where the blocks last added meet. The sides of this small cube, it will be seen, are each equal to the width of these blocks, that is, 2 feet. If each side is 2 feet long, the whole cube must contain 8 solid feet (because $2\times2\times2=8$), and it will be seen by Fig. 4, that this just fills the vacant corner, and completes the cube.

We have thus found, that the additions to be made around the large cube (Fig. 1) are as follows.

5400 solid feet upon three sides, (Fig. 2).
360 " " to fill the corners *a a a*.
8 " " to fill the deficiency in Fig. 3.

Fig. 4.



Now if these be added together, their sum will be 5768 solid feet, which subtracted from the dividend leave no remainder and the work is done. 32 feet is therefore the length of one side of the given cube.

The *proof* may be seen by involving the side now found to the third power, thus; $32\times32\times32=32768$; or it may be proved by adding together the contents of the several parts, thus,

27000 feet = contents of Fig. 1.
5400 " = addition to the three sides.
360 " = addition to fill the corners *a a a*.
8 " = addition to fill the corner in Fig. 3.

32768 Proof.

From these illustrations we see the reasons for the following rule.

RULE FOR EXTRACTING THE CUBE ROOT.

1. *Point off the given number, into periods of three figures each, beginning at the right.*

2. *Find the greatest cube in the left hand period, and subtract it from that period. Place the root in the quotient, and to the remainder bring down the next period, for a dividend.*

3. *Square the root already found (understanding a cipher at the right) and multiply it by 3 for a divisor.*

Divide the dividend by the divisor, and place the quotient for the next figure of the root.

4. *Multiply the divisor by this quotient figure. Multiply the square of this quotient figure by the former figure or figures of the root, and this product by three. Finally cube this quotient figure, and add these three results together for a subtrahend.*

5. *Subtract the subtrahend from the dividend. To the remainder bring down the next period, for a new dividend, and proceed as before.*

If it happens in any case, that the divisor is not contained in the dividend, or if there is a remainder after the last period is brought down, the same directions may be observed, that were given respecting the *square root*. (See page 218.)

EXAMPLES.

What is the cube root of 373248? 373248(72

343

$70^2 \times 3 =$ 14700)30248(First Dividend.

29400

$2^2 \times 70 \times 3 =$ 840

$2^3 =$ 8

30248 Subtrahend.

0000

Repeat the process of illustration. What is the rule for extracting the cube root?

Find the cube root of 941,192,000. A. 980. Of 958,585,256. A. 986. Of 478,211,768. A. 782. Of 494,913,671. A. 791. Of 445,943,744. A. 764. Of 196,122,941. A. 581. Of 204,336,469. A. 589. Of 57,512,456. A. 386. Of 6,751,269. A. 189. Of 39,651,821. A. 341. Of 42,508,549. A. 349. Of 510,082,399. A. 799. Of 469,097,433. A. 777.

Find the cube root of 7. A. 1.912933. Of 41. A. 3.448217. Of 49. A. 3.659306. Of 94. A. 4.546836. Of 97. A. 4.610436. Of 199. A. 5.838272. Of 179. A. 5.635741. Of 389. A. 7.299893. Of 364. A. 7.140037. Of 499. A. 7.931710. Of 699. A. 8.874809. Of 686. A. 8.819447. Of 886. A. 9.604569. Of 981. A. 9.936261.

The cube root of a *fraction*, is obtained by extracting the root of numerator and denominator, but if this cannot be done, it may be changed to a *decimal*, and the root extracted.

Find the cube root of $\frac{27}{1331}$. A. $\frac{3}{11}$. Of $\frac{13824}{29791}$. A. $\frac{24}{31}$. Of $\frac{456533}{970299}$. A. $\frac{77}{99}$. Of $\frac{7301384}{26730899}$. A. $\frac{194}{299}$. Of $\frac{20346417}{62570773}$. A. $\frac{273}{397}$.

Find the cube root of $\frac{5}{8}$. A. .8549879. Of $\frac{7}{40}$. A. .5593445. Of $\frac{24}{250}$. A. .4578857. Of $\frac{19}{200}$. A. .4562903. Of $\frac{124}{125}$. A. .9973262.

ARITHMETICAL PROGRESSION.

Any rank, or series of numbers, consisting of more than two terms, which increases or decreases by a common difference, is called an *Arithmetical series*, or *progression*.

When the series *increases*, that is, when it is formed by the constant *addition* of the common difference, it is called an ascending series, thus,

1, 3, 5, 7, 9, 11, &c.

Here it will be seen that the series is formed by a continual addition of 2 to each succeeding figure.

When the series decreases, that is, when it is formed by the constant *subtraction* of the common difference, it is called a descending series, thus,

14, 12, 10, 8, 6, 4, &c.

How is the cube root of a fraction obtained? When are numbers said to be in arithmetical progression?

Here the series is formed by a continual *subtraction* of 2, from each preceding figure.

The figures that make up the series are called the *terms* of the series. The *first* and *last* terms are called the *extremes*, and the other terms, the *means*.

From the above, it may be seen, that any term in a series may be found by continued addition or subtraction, but in a long series this process would be tedious. A much more expeditious method may be found.

1. The ages of *six* persons are in arithmetical progression. The youngest is 8 years old, and the common difference is 3, what is the age of the eldest? In other words, what is the last term of an arithmetical series, whose first term is 8, the number of terms 6, and the common difference 3?

8, 11, 14, 17, 20, 23.

Examining this series, we find that the common difference, 3, is added 5 times, that is *one less* than the number of terms, and the last term, 23, is larger than the first term, by five times the addition of the common difference, three; Hence the age of the elder person is $8 + 3 \times 5 = 23$.

Therefore when the first term, the number of terms, and the common difference, are given, to find the last term,

Multiply the common difference into the number of terms, less 1, *and add the product to the first term.*

2. If the first term be 4, the common difference 3, and the number of terms 100, what is the last term? Ans. 301.

3. There are, in a certain triangular field, 41 rows of corn; the first row, in 1 corner, is a single hill, the second contains 3 hills, and so on, with a common difference of 2; what is the number of hills in the last row? A. 81 hills.

4. A man puts out $1 at 6 per cent. simple interest, which, in 1 year, amounts to $1,06 in 2 years to $1,12, and so on, in arithmetical progression, with a common difference of $0,06; what would be the amount in 40 years? A. $3,40.

Hence we see, that the yearly amounts of any sum, at simple interest, form an arithmetical series; of which the *principal* is the *first term*, the *last amount* is the *last term*, the yearly *interest* is the *common difference*, and the number of *years* is 1 less than the *number* of terms.

It is often necessary to find the *sum of all the terms*, in an arithmetical progression. The most natural mode of obtaining the amount would be to add them together, but an easier method may be discovered, by attending to the following explanation.

1. Suppose we are required to find the sum of all the terms, in a series, whose first term is 2, the number of terms 10, and the common difference 2.

2,	4,	6,	8,	10,	12,	14,	16,	18,	20
20,	18,	16,	14,	12,	10,	8,	6,	4,	2
22,	22,	22,	22,	22,	22,	22,	22,	22,	22

The first row of figures above, represents the given series. The second, the same series with the order inverted, and the third, the sums of the additions of the corresponding terms in the two series. Examining these series, we shall find that the sums of the corresponding terms are the same, and that each of them is equal to the *sum of the extremes*, viz. 22. Now as there are 10 of these pairs in the two series, the sum of the terms in *both*, must be $22 \times 10 = 220$.

But it is evident, that the sum of the terms in *one* series, can be only *half* as great as the sum of *both*, therefore, if we divide 220 by 2, we shall find the sum of the terms in one series, which was the thing required. $220 \div 2 = 110$, the sum of the given series.

From this illustration we derive the following rule;

When the extremes and number of terms are given, to find the sum of the terms,

Multiply the sum of the extremes by the number of terms, and divide the product by 2.

2. The first term of a series is 1, the last term 29, and the number of terms 14. What is the sum of the series? A. 210.

3. 1st term, 2, last term, 51, number of terms, 18. Required the sum of the series. A. 477.

4. Find the sum of the natural terms 1, 2, 3, &c. to 10,000. A. 50,005,000.

5. A man rents a house for $50, annually, to be paid at the close of each year; what will the rent amount to in 20 years, allowing 6 per cent., simple interest, for the use of the money?

The last year's rent will evidently be $50 without interest, the last but *one* will be the amount of $50 for 1 year, the last but *two* the amount of $50 for 2 years, and so on, in arithmetical series, to the first, which will be the amount of $50 for 19 years = $107.

If the first term be 50, the last term 107, and the number of terms 20, what is the sum of the series? A. 1570.

6. What is the amount of an annual pension of $ 100, being in arrears, that is, remaining unpaid, for 40 years, allowing 5 per cent. *simple interest?* A. $ 7900.

7. There are, in a certain triangular field, 41 rows of corn; the first row, being in one corner, is a single hill, and the last row, on the side opposite, contains 81 hills; how many hills of corn in the field? A. 1681.

The method of finding the *common difference*, may be learned by what follows.

1. A man bought 100 yards of cloth in Arithmetical progression: for the first yard he gave 4 cents, and for the last 301 cents, what is the common increase on the price of each yard?

As he bought 100 yards, and at an increased price upon every yard, it is evident that this increase was made 99 times, or *once less* than the number of terms in the series. Hence the price of the last yard was greater than the first, by the addition of 99 times the regular increase.

Therefore if the first price be subtracted from the last, and the remainder be divided by the number of additions (99), the quotient will be the common increase; $301-4=297$ and $297 \div 99=3$, the common difference.

Hence, when the extremes and number of terms are given, to find the common difference,

Divide the difference of the extremes, by the number of terms less 1.

2. Extremes 3 and 19; number of terms 9. Required the common difference. A. 2.

3. Extremes 4 and 56; number of terms 14. Required the common difference. A. 4.

4. A man had 15 houses, increasing equally in value, from the first, worth $ 700, to the 15th, worth $ 3500. What was the difference in value between the first and second? A. 200.

In Arithmetical progression, any three of the following terms being given, the other two may be found. 1. The first term. 2. The last term. 3. The number of terms. 4. The common difference. 5. The sum of all the terms.

In arithmetical progression, what are the terms used, and what are the rules for finding them?

GEOMETRICAL PROGRESSION.

Any series of numbers, consisting of more than two terms, which increases by a common multiplier, or decreases by a common divisor, is called a *Geometrical Series.*

Thus the series 2, 4, 8, 16, 32, &c. consists of terms, each of which is *twice* the preceding, and this is an *increasing* or *ascending* Geometrical series.

The series 32, 16, 8, 4, 2, consists of numbers, each of which is *one half* the preceding, and this is a *decreasing* or *descending* Geometrical series.

The common multiplier or divisor is called the *Ratio*, and the numbers which form the series are called *Terms.*

As in Arithmetical, so in Geometrical progression, if any three of the five following terms be given, the other two may be found.

1. The first term. 2. The last term. 3. The number of terms. 4. The common difference. 5. The sum of all the terms.

1. A man bought a piece of cloth containing 12 yards, the first yard cost 3 cents, the second 6, the third 12, and so on, doubling the price to the last, what cost the last yard?

$3\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2=3\times2^{11}=6144$ Ans.

In examining the above process, it will be seen, that the price of the *second* yard is found by multiplying the first payment into the ratio (2) *once;* the price of the *third* yard, by multiplying by 2 *twice*, &c., and that the ratio (2) is used as a factor *eleven* times, or *once less* than the number of terms. The last term then, is the *eleventh* power of the ratio (2) multiplied by the first term (3).

Hence the first term, ratio, and number of terms, being given, to find the *last* term.

Multiply the first term, by that power of the ratio, whose index is one less than the number of terms.

NOTE. In involving the ratio, it is not always necessary to produce all the intermediate powers; the process may often be abridged, by multiplying together two powers already obtained, thus,

The 11th power = the 6th power × the 5th power, &c.

When are numbers in geometrical progression? What are the terms used? What are the rules for finding them?

2. If the first term is 2, the ratio 2, and the number of terms 13, what is the last term? A. 8,192.

3. Find the 12th term of a series, whose first term is 3, and ratio, 3. A. 531,441.

4. A man plants 4 kernels of corn, which, at harvest, produce 32 kernels; these he plants the second year; now, supposing the annual increase to continue 8 fold, what would be the produce of the 16th year, allowing 1000 kernels to a pint?

A. 2199023255.552 bushels.

5. Suppose a man had put out one cent at compound interest in 1620, what would have been the amount in 1824, allowing it to double once in 12 years?

$2^{17} = 131072$. A. 1310.72.

The most obvious method of obtaining the *sum of the terms* in a Geometrical series, might be by *addition*, but this is not the most expeditious, as will be seen.

1. A man bought 5 yards of cloth, giving 2 cents for the first, 6 cents for the second, and so in 3 fold ratio; what did the whole cost him?

2, 6, 18, 54, 162
6, 18, 54, 162, 486

The first of the above lines, represents the original series. The second, that series, multiplied by the ratio 3.

Examining these series, it will be seen that their terms are all alike excepting two: viz. the *first* term of the first series, and the *last* of the second series. If now we subtract the first series from the last, we have for a remainder $486 - 2 = 484$, as all the intermediate terms vanish in the subtraction.

Now the last series is *three* times the first, (for it was made by multiplying the first series by 3,) and as we have already subtracted *once* the first, the remainder must of course be *twice* the first.

Therefore if we divide 484 by 2, we shall obtain the sum of the first series. $584 \div 2 = 242$ Ans.

As in the preceding process, all the terms vanish in the subtraction, excepting the first and last, it will be seen, that the result would have been the same, if the last term only, had been multiplied, and the first subtracted from the product.

Hence, the extremes and ratio being given, to find the sum of all the terms,

Multiply the greater term by the ratio, from the product subtract the least term, and divide the remainder by the ratio less 1.

2. Given the first term, 1; the last term, 2,187; and the ratio, 3; required the sum of the series. A. 3,280.

3. Extremes, 1 and 65,536; ratio 4; required the sum of the series. A. 87,381.

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5. What is the sum of the series 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, and so on, to an infinite extent? A. 21$\frac{1}{3}$.

Here it is evident, the last term is 0, or indefinitely near to nothing, the extremes therefore are 16 and 0, and the ratio 4.

ANNUITIES.

An annuity is a sum payable periodically, for a certain length of time, or for ever.

An annuity, in the proper sense of the word, is a sum paid *annually*, yet payments made at different periods, are called annuities. Pensions, rents, salaries, &c. belong to annuities.

When annuities are not paid at the time they become due, they are said to be in *arrears*.

The sum of all the annuities in arrears, with the interest on each for the time they have remained due, is called the *amount*.

The *present worth* of an annuity, is the sum which should be paid for an annuity *yet to come*.

When an annuity is to continue for ever, its present worth is a sum, whose yearly interest equals the annuity.

Now as the principal, multiplied by the rate, will give the interest, the interest, divided by the rate, will give the principal.

Hence to find the *present worth* of an annuity, continuing for ever.

Divide the annuity by the rate per cent.

1. What is the worth of $100 annuity, to continue for ever, allowing to the purchaser 4 per cent.? allowing 5 per cent.? 8 per cent.? 10 per cent.? 15 per cent.? 20 per cent.? Ans. to last, $500.

2. What is an estate worth, which brings in $7,500 a year, allowing 6 per cent.? A. $125,000.

What is an annuity? When is an annuity in arrears? What is the amount? What is the present worth of an annuity? What is the rule to find the present worth?

ANNUITIES AT COMPOUND INTEREST.

It has been shown (page 188) that Compound Interest is that which arises from adding the interest to the principal at the close of each year, and making the amount a new principal. The amount of $ 1 for one year at 6 per cent. is $1.06, and it will be found, that if the principal be multiplied by this, the product will be the amount for 1 year, and this amount multiplied by 1.06, will be the amount for 2 years, and so on. Hence we see that any sum at compound interest, forms a geometrical series, of which the *ratio* is the amount of $1 at the given rate per cent.

1. An annuity of $40 was left 5 years unpaid, what was then due upon it, allowing 5 per cent. compound interest?

It is evident that for the *fifth* or *last* year, the annuity alone is due; for the *fourth*, the amount of the annuity for 1 year; for the third the amount of the annuity for 2 years, and so on; and the *sum* of these amounts will be the answer, or what is due in 5 years,

From this we find that the amount of an annuity in arrears, forms a geometrical progression, whose *first term* is the annuity, the *ratio*, the amount of $ 1 at the given rate, and the *number of terms*, the number of years.

The above example, then, may be resolved into the following question. What is the sum of a geometrical series whose first term is $40, the ratio 1.05, and the number of terms 5? First find the *last* term, by the first rule in Geometrical progression, and then the *sum* of the series by the second rule. The answer will be found to be $221.02.

Hence, to find the amount of an annuity in arrears, at compound interest,

Find the sum of a Geometrical series, whose first term is the annuity, whose ratio, the amount of $ 1 at the given rate per cent., and whose number of terms is the number of years.

NOTE. A table, showing the amount of $1 at 5 and 6 per cent., compound interest, for any number of years not exceeding 24, will be found on page 189.

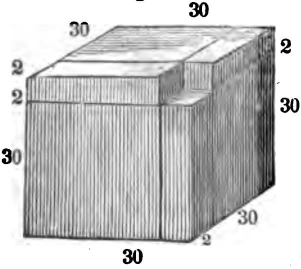
2. What is the amount of an annuity of $50, it being *in arrears* 20 years, allowing 5 per cent. compound interest?

A. $1653,29.

2 feet; $2 \times 2 = 4$, and this multiplied by 30 (the *length*) $= 120$, which is the solid contents of *one* block. But in *three*, there will be three times as many solid feet, or 360, which is the number required to fill the deficiencies.

In other words, we square the last quotient figure (2) multiply the product by the first figure of the quotient (3 tens) and then multiply the last product by 3, the number of deficiencies.

Fig. 3.

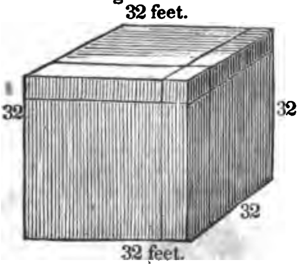


But by examining Fig. 3, it appears that the figure is not yet complete, but that a small cube is still wanting, where the blocks last added meet. The sides of this small cube, it will be seen, are each equal to the width of these blocks, that is, 2 feet. If each side is 2 feet long, the whole cube must contain 8 solid feet (because $2 \times 2 \times 2 = 8$), and it will be seen by Fig. 4, that this just fills the vacant corner, and completes the cube.

We have thus found, that the additions to be made around the large cube (Fig. 1) are as follows.

5400 solid feet upon three sides, (Fig. 2).
360 " " to fill the corners *a a a*.
8 " " to fill the deficiency in Fig. 3.

Fig. 4.



Now if these be added together, their sum will be 5768 solid feet, which subtracted from the dividend leave no remainder and the work is done: 32 feet is therefore the length of one side of the given cube.

The *proof* may be seen by involving the side now found to the third power, thus; $32 \times 32 \times 32 = 32768$; or it may be proved by adding together the contents of the several parts, thus,

27000 feet = contents of Fig. 1.
5400 " = addition to the three sides.
360 " = addition to fill the corners *a a a*.
8 " = addition to fill the corner in Fig. 3.

32768 Proof.

From these illustrations we see the reasons for the following rule.

RULE FOR EXTRACTING THE CUBE ROOT.

1. *Point off the given number, into periods of three figures each, beginning at the right.*

2. *Find the greatest cube in the left hand period, and subtract it from that period. Place the root in the quotient, and to the remainder bring down the next period, for a dividend.*

3. *Square the root already found (understanding a cipher at the right) and multiply it by 3 for a divisor.*

Divide the dividend by the divisor, and place the quotient for the next figure of the root.

4. *Multiply the divisor by this quotient figure. Multiply the square of this quotient figure by the former figure or figures of the root, and this product by three. Finally cube this quotient figure, and add these three results together for a subtrahend.*

5. *Subtract the subtrahend from the dividend. To the remainder bring down the next period, for a new dividend, and proceed as before.*

If it happens in any case, that the divisor is not contained in the dividend, or if there is a remainder after the last period is brought down, the same directions may be observed, that were given respecting the *square root*. (See page 218.)

EXAMPLES.

What is the cube root of 373248? 373248(72

343

$70^2 \times 3 = 14700$)30248(First Dividend.

29400

$2^2 \times 70 \times 3 =$ 840

$2^3 =$ 8

30248 Subtrahend.

0000

Repeat the process of illustration. What is the rule for extracting the cube root?

Find the cube root of 941,192,000. A. 980. Of 958,585,256. A. 986. Of 478,211,768. A. 782. Of 494,913,671. A. 791. Of 445,943,744. A. 764. Of 196,122,941. A. 581. Of 204,336,469. A. 589. Of 57,512,456. A. 386. Of 6,751,269. A. 189. Of 39,651,821. A. 341. Of 42,508,549. A. 349. Of 510,082,399. A. 799. Of 469,097,433. A. 777.

Find the cube root of 7. A. 1.912933. Of 41. A. 3.448217. Of 49. A. 3.659306. Of 94. A. 4.546836. Of 97. A. 4.610436. Of 199. A. 5.838272. Of 179. A. 5.635741. Of 389. A. 7.299893. Of 364. A. 7.140037. Of 499. A. 7.931710. Of 699. A. 8.874809. Of 686. A. 8.819447. Of 886. A. 9.604569. Of 981. A. 9.936261.

The cube root of a *fraction*, is obtained by extracting the root of numerator and denominator, but if this cannot be done, it may be changed to a *decimal*, and the root extracted.

Find the cube root of $\frac{27}{1331}$. A. $\frac{3}{11}$. Of $\frac{13824}{29791}$. A. $\frac{24}{31}$. Of $\frac{456533}{970299}$. A. $\frac{77}{99}$. Of $\frac{7301384}{26730899}$. A. $\frac{194}{299}$. Of $\frac{20346417}{62570773}$. A. $\frac{273}{397}$.

Find the cube root of $\frac{5}{8}$. A. .8549879. Of $\frac{7}{40}$. A. .5593445. Of $\frac{24}{250}$. A. .4578857. Of $\frac{19}{200}$. A. .4562903. Of $\frac{124}{125}$. A. .9973262.

ARITHMETICAL PROGRESSION.

Any rank, or series of numbers, consisting of more than two terms, which increases or decreases by a common difference, is called an *Arithmetical series*, or *progression*.

When the series *increases*, that is, when it is formed by the constant *addition* of the common difference, it is called an ascending series, thus,

1, 3, 5, 7, 9, 11, &c.

Here it will be seen that the series is formed by a continual addition of 2 to each succeeding figure.

When the series decreases, that is, when it is formed by the constant *subtraction* of the common difference, it is called a descending series, thus,

14, 12, 10, 8, 6, 4, &c.

How is the cube root of a fraction obtained? When are numbers said to be in arithmetical progression?

Here the series is formed by a continual *subtraction* of 2, from each preceding figure.

The figures that make up the series are called the *terms* of the series. The *first* and *last* terms are called the *extremes*, and the other terms, the *means*.

From the above, it may be seen, that any term in a series may be found by continued addition or subtraction, but in a long series this process would be tedious. A much more expeditious method may be found.

1. The ages of *six* persons are in arithmetical progression. The youngest is 8 years old, and the common difference is 3, what is the age of the eldest? In other words, what is the last term of an arithmetical series, whose first term is 8, the number of terms 6, and the common difference 3?

8, 11, 14, 17, 20, 23.

Examining this series, we find that the common difference, 3, is added 5 times, that is *one less* than the number of terms, and the last term, 23, is larger than the first term, by five times the addition of the common difference, three; Hence the age of the elder person is $8 + 3 \times 5 = 23$.

Therefore when the first term, the number of terms, and the common difference, are given, to find the last term,

Multiply the common difference into the number of terms, less 1, *and add the product to the first term.*

2. If the first term be 4, the common difference 3, and the number of terms 100, what is the last term? Ans. 301.

3. There are, in a certain triangular field, 41 rows of corn; the first row, in 1 corner, is a single hill, the second contains 3 hills, and so on, with a common difference of 2; what is the number of hills in the last row? A. 81 hills.

4. A man puts out $ 1 at 6 per cent. simple interest, which, in 1 year, amounts to $ 1,06 in 2 years to $ 1,12, and so on, in arithmetical progression, with a common difference of $ 0 ,06; what would be the amount in 40 years? A. $ 3 ,40.

Hence we see, that the yearly amounts of any sum, at simple interest, form an arithmetical series, of which the *principal* is the *first term*, the *last amount* is the *last term*, the yearly *interest* is the *common difference*, and the number of *years* is 1 less than the *number* of terms.

It is often necessary to find the *sum of all the terms*, in an arithmetical progression. The most natural mode of obtaining the amount would be to add them together, but an easier method may be discovered, by attending to the following explanation.

1. Suppose we are required to find the sum of all the terms, in a series, whose first term is 2, the number of terms 10, and the common difference 2.

2,	4,	6,	8,	10,	12,	14,	16,	18,	20
20,	18,	16,	14,	12,	10,	8,	6,	4,	2
22,	22,	22,	22,	22,	22,	22,	22,	22,	22

The first row of figures above, represents the given series. The second, the same series with the order inverted, and the third, the sums of the additions of the corresponding terms in the two series. Examining these series, we shall find that the sums of the corresponding terms are the same, and that each of them is equal to the *sum of the extremes*, viz. 22. Now as there are 10 of these pairs in the two series, the sum of the terms in *both*, must be $22 \times 10 = 220$.

But it is evident, that the sum of the terms in *one* series, can be only *half* as great as the sum of *both*, therefore, if we divide 220 by 2, we shall find the sum of the terms in one series, which was the thing required. $220 \div 2 = 110$, the sum of the given series.

From this illustration we derive the following rule;

When the extremes and number of terms are given, to find the sum of the terms,

Multiply the sum of the extremes by the number of terms, and divide the product by 2.

2. The first term of a series is 1, the last term 29, and the number of terms 14. What is the sum of the series? A. 210.

3. 1st term, 2, last term, 51, number of terms, 18. Required the sum of the series. A. 477.

4. Find the sum of the natural terms 1, 2, 3, &c. to 10,000. A. 50,005,000.

5. A man rents a house for $ 50, annually, to be paid at the close of each year; what will the rent amount to in 20 years, allowing 6 per cent., simple interest, for the use of the money?

The last year's rent will evidently be $ 50 without interest, the last but *one* will be the amount of $ 50 for 1 year, the last but *two* the amount of $ 50 for 2 years, and so on, in arithmetical series, to the first, which will be the amount of $ 50 for 19 years = $ 107.

If the first term be 50, the last term 107, and the number of terms 20, what is the sum of the series? A. 1570.

6. What is the amount of an annual pension of $100, being in arrears, that is, remaining unpaid, for 40 years, allowing 5 per cent. *simple interest?* A. $7900.

7. There are, in a certain triangular field, 41 rows of corn; the first row, being in one corner, is a single hill, and the last row, on the side opposite, contains 81 hills; how many hills of corn in the field? A. 1681.

The method of finding the *common difference*, may be learned by what follows.

1. A man bought 100 yards of cloth in Arithmetical progression: for the first yard he gave 4 cents, and for the last 301 cents, what is the common increase on the price of each yard?

As he bought 100 yards, and at an increased price upon every yard, it is evident that this increase was made 99 times, or *once less* than the number of terms in the series. Hence the price of the last yard was greater than the first, by the addition of 99 times the regular increase.

Therefore if the first price be subtracted from the last, and the remainder be divided by the number of additions (99), the quotient will be the common increase; $301-4=297$ and $297\div 99=3$, the common difference.

Hence, when the extremes and number of terms are given, to find the common difference,

Divide the difference of the extremes, by the number of terms less 1.

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The common multiplier or divisor is called the *Ratio*, and the numbers which form the series are called *Terms.*

As in Arithmetical, so in Geometrical progression, if any three of the five following terms be given, the other two may be found.

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Hence to find the *present worth* of an annuity, continuing for ever.

Divide the annuity by the rate per cent.

1. What is the worth of $100 annuity, to continue for ever, allowing to the purchaser 4 per cent.? allowing 5 per cent.? 8 per cent.? 10 per cent.? 15 per cent.? 20 per cent.? Ans. to last, $500.

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1. An annuity of $40 was left 5 years unpaid, what was then due upon it, allowing 5 per cent. compound interest?

It is evident that for the *fifth* or *last* year, the annuity alone is due; for the *fourth*, the amount of the annuity for 1 year; for the third the amount of the annuity for 2 years, and so on; and the *sum* of these amounts will be the answer, or what is due in 5 years,

From this we find that the amount of an annuity in arrears, forms a geometrical progression, whose *first term* is the annuity, the *ratio*, the amount of $ 1 at the given rate, and the *number of terms*, the number of years.

The above example, then, may be resolved into the following question. What is the sum of a geometrical series whose first term is $40, the ratio 1.05, and the number of terms 5? First find the *last* term, by the first rule in Geometrical progression, and then the *sum* of the series by the second rule. The answer will be found to be $221.02.

Hence, to find the amount of an annuity in arrears, at compound interest,

Find the sum of a Geometrical series, whose first term is the annuity, whose ratio, the amount of $ 1 at the given rate per cent., and whose number of terms is the number of years.

NOTE. A table, showing the amount of $1 at 5 and 6 per cent., compound interest, for any number of years not exceeding 24, will be found on page 189.

2. What is the amount of an annuity of $50, it being *in arrears* 20 years, allowing 5 per cent. compound interest?

A. $1653,29.

3. If the annual rent of a house, which is $ 150, be in arrears 4 years, what is the amount, allowing 10 per cent. compound interest? A. $ 696,15.

4. To how much would a salary of $ 500 per annum amount in 14 years, the money being improved at 6 per cent., compound interest? in 10 years? in 20 years? in 22 years? in 24 years? Ans. to the last, $ 25,407,75.

5. Find the amount of an annuity of $ 150, for 3 years, at 6 per cent. A. $ 477,54.

A rule has been given, for finding the *present worth* of an annuity, to continue for ever; but it is often necessary to find the present worth of an annuity, which is to continue for a limited number of years; thus,

6. What is the present worth of an annual pension of $ 100 to continue 4 years, allowing 6 per cent. compound interest?

The present worth is evidently a sum, which, at compound interest, would in 4 years produce an amount equal to the *amount of the annuity*, for the same time.

Now to find a given amount, at compound interest, we multiply a sum by the amount of $ 1 at the given rate per cent. as many times successively as there are years.

Hence, to find a sum, which will produce a given amount in a certain time, we must *reverse* this process and *divide* by the amount of $ 1 for the given time.

Applying this to the above example, we find by the preceding rule, that the amount is $ 437,46. Dividing this by the amount of $ 1 for 4 years, we find the present worth,

437,46 ÷ 1,26247 = $ 346,511, Ans.

Hence to find the present worth of an annuity,

Find the amount in arrears for the whole time, and divide it by the amount of $ 1 at the given rate per cent., for the given number of years.

The operations under this rule, will be facilitated by the following TABLE, showing the present worth of $ 1, or £ 1 annuity, at 5 and 6 per cent. compound interest, for any number of years from 1 to 34.

What is the rule for finding the amount of an annuity? What is the rule for finding the present worth of an annuity?

TABLE.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0,95238	0,94339	18	11,68958	10,8276
2	1,85941	1,83339	19	12,08532	11,15811
3	2,72325	2,67301	20	12,46221	11,46992
4	3,54595	3,4651	21	12,82115	11,76407
5	4,32948	4,21236	22	13,163	12,04158
6	5,07569	4,91732	23	13,48807	12,30338
7	5,78637	5,58238	24	13,79864	12,55035
8	6,46321	6,20979	25	14,09394	12,78335
9	7,10782	6,80169	26	14,37518	13,00316
10	7,72173	7,36008	27	14,64303	13,21053
11	8,30641	7,88687	28	14,89813	13,40616
12	8,86325	8,38384	29	15,14107	13,59072
13	9,39357	8,85268	30	15,37245	13,76483
14	9,89864	9,29498	31	15,59281	13,92908
15	10,37966	9,71225	32	15,80268	14,08398
16	10,83777	10,10589	33	16,00255	14,22917
17	11,27407	10,47726	34	16,1929	14,36613

It is evident, that the present worth of $ 2 annuity is 2 times as much as that of $ 1; the present worth of $ 3 will be 3 times as much, &c. Hence, *to find the present worth of any annuity, at* 5 *or* 6 *per cent.*,—Find, in this table, the present worth of $ 1 annuity, and multiply it by the *given annuity*, and the product will be the *present worth*.

7. Find the present worth of a $ 40 annuity, to continue 5 years, at 5 per cent. A. $ 173.173.

8. Find the present worth of $ 100 annuity, for 20 years, at 5 per cent. A. $ 1,246.22.

9. Find the present worth of an annuity of $ 21,54 for 7 years at 6 per cent. A. 120.244+

10. Find the present worth of an annuity of $ 100, to continue 12 years, at 6 per cent. A. $ 838.384.

11. Find the present worth of an annuity of $ 936, for 20 years, at 5 per cent. A. $ 11,664.629—

As the present worth of any annuity may be found, by multiplying the annuity by one of the numbers, in the above table, it is plain that if any *present worth* be *divided* by the same number, it will give the *annuity itself*.

Hence to discover of what annuity any given sum is the present worth, we may use the above, as a table of *divisors*, instead of multipliers.

What annuity to continue 19 years, will $ 6,694.866 purchase, when money will bring 6 per cent.? A. $ 600.

An annuity is said to be *in reversion*, when it does not commence until some future time.

12. What is the present worth of $ 60 annuity, to be continued 6 years, but not to commence till 3 years hence, allowing 6 per cent. compound interest?

The present worth is evidently such a sum as would in 3 years, at six per cent., compound interest, produce an amount, equal to the present worth of the annuity, were it to commence immediately.

We must therefore first find the present worth of an annuity of $ 60 to commence immediately, according to the last rule. This we shall discover to be $ 295.039.

We now wish to obtain a sum, whose amount in 3 years will equal this present worth. This may be found by dividing the $ 295.039 by the *amount* of $ 1 for 3 years thus,

$ 295.039÷1,19101=247.72. Ans. $ 247,72.

Hence to find the present worth of any annuity taken in reversion, at compound interest,

Find the present worth to commence immediately, and this sum divided by the amount of $ 1 for the time in reversion, will give the answer.

13. What is the present worth of a lease of $ 100 to continue 20 years, but not to commence till the end of 4 years, allowing 5 per cent.? what if it be 6 years in reversion? 8 years? 10 years? 14 years? Ans. to last, $ 629,426.

14. What is the present worth of $ 100 annuity, to be continued 4 years, but not to commence till 2 years hence, allowing 6 per cent. compound interest? A. $ 308,393.

PERMUTATION.

Permutation is the method of finding how many changes may be made, in the order in which things succeed each other.

What number of permutations may be made on the letters A and B? They may be written A B, or B A.

What number on the letters A B C?

Placing A first, A B C, or A C B.

Placing B first, B A C, or B C A.

When is an annuity said to be in reversion? What is the rule for finding the present worth of an annuity taken in reversion? What is permutation. What is the rule?

Placing C first, C A B, or C B A.

From these examples it will be seen, that of two things, there may be 2 changes, ($1\times2=2$,) and of three things there may be 6 changes, ($1\times2\times3=6$.)

Hence, to find the number of different changes, or permutations, of which any number of different things are capable,

Find the continual product of the natural series of numbers, from 1 *to the given number.*

1. Four gentlemen agreed to remain together, as long as they could arrange themselves differently at dinner. How many days did they remain? A. 24 days.

2. 10 gentlemen made the same agreement, but they all died before it could be fulfilled. The last survivor lived 53 yrs. 98 days, after the agreement. How much did the bargain then want of being fulfilled, allowing 365 days to the year?
A. 9,888 yrs. 237 d.

3. How many years will it take to ring all the possible changes on 12 bells, supposing that 10 can be rung in a minute, and that the year contains 365 d. 5 h. 49 m? A. 91 yrs. 26 d. 22h. 41 m.

4. How many variations may there be in the position of the nine digits? Ans. 362880.

5. A man bought 25 cows, agreeing to pay for them 1 cent for every different order in which they could all be placed; how much did the cows cost him? Ans. $ 155112100433309859840000.

MISCELLANEOUS EXAMPLES.

Many of these sums are designed for mental exercise. In solving the first 50, the pupil should not be allowed to use the slate.

1. If two men start from the same place and travel in opposite directions, one at the rate of $4\frac{5}{7}$ miles an hour, and the other at the rate of $3\frac{2}{5}$ miles an hour, how far will they be apart in 6 hours?

2. If 6 bushels of oats will keep 3 horses a week, how many bushels will be required to keep 12 horses the same time?

3. If you give 5 men $3\frac{1}{5}$ bushels of corn apiece, how much do you give the whole?

4. If 8 dollars worth of provisions will serve 9 men 5 days, how many days will it serve 12 men? how many days would it serve 3 men?

5. If $6 worth of provision will serve 5 men 8 days, how many days would it serve 9 men? how many days would it serve 3 men?

6. If $12 worth of provision would serve 5 man 7 days, how many men would it serve 9 days?

7. If one peck of wheat afford 9 six penny loaves, how many ten penny loaves would it afford?

8. If a man paid $60 to his laborers, giving to every man 9d. and to every boy 3d. if the men and boys were equal in number, how many were there of each?

9. Two men bought a barrel of flour together, one paid $3 and the other paid $5; what part of the whole did each pay, and what part of the barrel ought each to have?

10. Three men hired a field together, A paid $7, B paid $3, and C paid $8, what part of the whole did each pay, and what part of the produce ought each to have?

11. Three men bought a lottery ticket together, A paid $6, B paid $4, and C paid $10. They drew a prize of $150, what was each man's share?

12. Three men hired a pasture together for $60. A put in 2 horses, B 4 horses, and C 6 horses, how much ought each to pay?

13. Three men commenced trade together, and advanced money in this proportion—For every $5 that A put in, B put in $3, and C put in $2, they gained $100, what was each man's share?

14. Two men hired a pasture for $32. A put in 3 sheep for 4 months, and B put in 4 sheep for 5 months, how much ought each to pay?

NOTE. 3 sheep for 4 months is the same as 12 sheep for *one* month, and 4 sheep for 5 months is the same as 20 sheep for one month.

15. A and B traded together and invested money in the following proportions, A put in $10 for 2 months, and B put in $5 for 3 months. They gained $70; what was each man's share?

16. Three men traded in company, and put in money in the following proportions. A put in 4 dollars as often as B put in 3, and as often as C put in 2. A's money was in 2 months, B's 3 months, and C's 4 months. They gained $100; what was each man's share?

17. Two men traded in company. A put in $2 as often as B

put in $3. A's money was employed 7 months, and B's 5 months. They gained 58 dollars. What was each man's share?

18. If A can do $\frac{1}{2}$ of a piece of work in 1 day, and B can do $\frac{1}{4}$ of it in one day, how much would both do in a day? How long would it take them both together to do the whole?

19. If 1 man can do a piece of work in 2 days, and another in 3 days, how much of it would each do in a day? How much would both together do? How long would it take them both to do the whole?

20. A cistern has 2 cocks; the first will fill it in 3 hours, the second in 6 hours; how much of it would each fill in an hour? How much would both together fill? How long would it take them both to fill it?

21. A man and his wife found by experience, that, when they were both together, a bushel of meal would last them only 2 weeks; but when the man was gone, it would last his wife 5 weeks. How much of it did both together consume in 1 week? What part did the woman alone consume in 1 week? What part did the man alone consume in 1 week? How long would it last the man alone?

22. If 1 man could build a piece of wall in 5 days, and another man could do it in 7 days, how much of it would each do in 1 day? How many days would it take them both to do it?

23. A cistern has 3 cocks; the first would fill it in 3 hours, the second in 6 hours; the third in 4 hours; what part of the whole would each fill in 1 hour? and how long would it take them all to fill it, if they were all running at once?

24. A and B together can build a boat in 8 days, and with the assistance of C they can do it in 5 days; how much of it can A and B build in 1 day? How much of it can A, B, and C, build in 1 day? How much of it can C build alone in 1 day? How long will it take C to build it alone?

25. Suppose I would line 8 yards of broadcloth that is $1\frac{1}{2}$ yards wide, with shalloon that is $\frac{3}{4}$ of a yard wide; how many yards of the shalloon will line 1 yard of the broadcloth? How many yards will line the whole?

26. If 7 yards of cloth cost 13 dollars, what will 10 yards cost?

27. If the wages of 25 weeks come to 75 dollars, what will be the wages of seven weeks?

28. If 8 tons of hay will keep 7 horses three months, how much will keep 12 horses the same time?

29. If a staff 4 feet long cast a shadow 6 feet long, what is the

length of a pole that casts a shadow 58 feet at the same time of day.

30. If a stick 8 feet long cast a shadow 2 feet in length, what is the height of a tree which casts a shadow 42 feet at the same time of day?

31. A ship has sailed 24 miles in 4 hours; how long will it take her to sail 150 at the same rate?

32. 30 men can perform a piece of work in 20 days; how many men will it take to perform the same work in 8 days?

33. 17 men can perform a piece of work in 25 days; in how many days would 5 men perform the same work?

34. A hare has 76 rods the start of a greyhound, but the greyhound runs 15 rods to 10 of the hare; how many rods must the greyhound run to overtake the hare?

35. A garrison has provision for 8 months, at the rate of 15 ounces per day; how much must be allowed per day, in order that the provision may last 11 months?

36. If 8 men can build a wall 15 rods in length in 10 days, how many men will it take to build a wall 45 rods in length in 5 days?

37. A man being asked the price of his horse, answered, that his horse and saddle together were worth 100 dollars; but the horse was worth 9 times as much as the saddle. What was each worth?

38. A man having a horse, a cow, and a sheep, was asked what was the value of each. He answered that the cow was worth twice as much as the sheep, and the horse 3 times as much as the sheep, and that all together were worth 60 dollars. What was the value of each?

39. If 80 dollars worth of provision will serve 20 men 24 days, how many days will 100 dollars worth of provision serve 30 men?

40. The third part of an army was killed, the fourth part taken prisoners, and 1000 fled; how many were in this army?

This, and the following 10 questions, are usually classed under the rule of *Position*, but they may be solved in a much more simple and easy manner. Thus, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of the army. Now as there are 12 twelfths in the whole, 1000 must be the remaining 5 twelfths. If 1000 is 5 twelfths of the army, 1 fifth of 1000, or 200, will be 1 twelfth; and if 200 is 1 twelfth, the whole, or 12 twelfths will be 12 times as much, or 2400.

41. A farmer being asked how many sheep he had, answered, that he had them in 4 pastures; in the first he had $\frac{1}{3}$ of his flock; in the second $\frac{1}{4}$; in the third $\frac{1}{6}$; and in the fourth 15; how many sheep had he?

42. A man driving his geese to market, was met by another, who said, good morrow, master, with your hundred geese; says he, I have not a hundred; but if I had half as many more as I now have, and two geese and a half, I should have a hundred; how many had he?

43. What number is that, to which if its half be added, the sum will be 60?

44. What number is that, to which if its third be added the sum will be 48?

45. What number is that, to which if its 5th be added the sum will be 54?

46. What number is that to which if its half and its third be added the sum will be 55?

47. A man being asked his age, answered, that if its half and its third were added to it, the sum would be 77; what was his age?

48. What number is that, which being increased by its half, its fourth, and eighteen more, will be doubled?

49. A boy being asked his age, answered, that if $\frac{1}{2}$ and $\frac{1}{4}$ of his age, and 20 more were added to his age, the sum would be 3 times his age. What was his age?

50. A man being asked how many sheep he had, answered, that if he had as many more, $\frac{1}{2}$ as many more, and $2\frac{1}{2}$ sheep, he should have 100. How many had he?

51. A farmer carried his grain to market, and sold

75 bushels of wheat, at \$ 1,45 per bushel,
64 " rye, " \$,95 " "
142 " corn, " \$,50 " "

In exchange he received sundry articles:—

3 pieces of cloth, each
containing 31 yds., at \$ 1,75 per yd.
2 quintals of fish, " \$ 2,30 per quin.
8 hhds of salt, " \$ 4,30 per hhd.

and the balance in money.

How much money did he receive? Ans. \$ 38,80.

52. A man exchanges 760 gallons of molasses, at $37\frac{1}{2}$ cents

per gallon, for 66½ cwt. of cheese, at $4 per cwt.; how much will be the balance in his favor? Ans. $19.

53. Bought 84 yards of cloth, at $1,25 per yard; how much did it come to? How many bushels of wheat, at $1,50 per bushel, will it take to pay for it? Ans. to the last, 70 bushels.

54. A man sold 342 pounds of beef, at 6 cents per pound, and received his pay in molasses, at 37½ cents per gallon; how many gallons did he receive? Ans. 54,72 gallons.

55. A man exchanged 70 bushels of rye, at $,92 per bushel, for 40 bushels of wheat, at $1,37½ per bushel, and received the balance in oats, at $,40 per bushel; how many bushels of oats did he receive? Ans. 23½.

56. How many bushels of potatoes, at 1s. 6d. per bushel, must be given for 32 bushels of barley, at 2s. 6d. per bushel?

Ans. 53⅓ bushels.

57. How much salt, at $1,50 per bushel, must be given in exchange for 15 bushels of oats, at 2s. 3d. per bushel?

Note. It will be recollected that, when the price and cost are given, to find the quantity, they must both be reduced to the same denomination before dividing. Ans. 3¾ bushels.

58. How much wine, at $2,75 per gallon, must be given in exchange for 40 yards of cloth, at 7s. 6d. per yard?

Ans. $18\frac{2}{11}$ gallons.

59. There is a fish, whose head is 4 feet long; his tail is as long as his head and ½ the length of his body, and his body is as long as his head and tail; what is the length of the fish?

The pupil will perceive that the length of the body is ½ the length of the fish. Ans. 32 feet.

60. A gentleman had 7 £. 17s. 6d. to pay among his laborers; to every boy he gave 6d., to every woman 8d., and to every man 16d.; and there were for every boy three women, and for every woman two men; I demand the number of each.

Ans. 15 boys, 45 women, and 90 men.

61. A farmer bought a sheep, a cow, and a yoke of oxen for $82,50; he gave for the cow 8 times as much as for the sheep, and for the oxen 3 times as much as for the cow; how much did he give for each?

Ans. For the sheep $2,50, the cow $20, and the oxen $60.

62. There was a farm, of which A owned $\frac{2}{7}$, and B $\frac{15}{21}$; the farm was sold for $1764; what was each one's share of the money? Ans. A's $504, and B's $1260.

63. Four men traded together on a capital of $ 3000, of which A put in $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{12}$; at the end of three yrs., they had gained $ 2364; what was each one's share of the gain?

Ans. { A's $ 1182
B's $ 591
C's $ 394
D's $ 197 }

64. Bought a book, the price of which was marked $ 4,50, but for cash the bookseller would sell it at 33⅓ per cent. discount; what is the cash price? Ans. $ 3,00.

65. A merchant bought a cask of molasses, containing 120 gallons, for $ 42; for how much must he sell it to gain 15 per cent.? How much per gallon? Ans. to last, $,40¼.

66. A merchant bought a cask of sugar, containing 740 pounds, for $ 59,20; how must he sell it per pound to gain 25 per cent? Ans. $,10.

67. What is the interest, at 6 per cent., of $ 71,02 for 17 months 12 days? Ans. $ 6,178+

68. What is the interest of $ 487,003 for 18 months? Ans. $ 43,83+

It has been shown that the length of one side of a square multiplied into itself, will give the square contents.

Hence to find the area, or superficial contents of a square when one side is given,

Multiply the side of the square into itself.

69. There is a room 18 feet square; how many yards of carpeting 1 yard wide will cover it?

Ans. $18^2 = 324$ft. $= 36$ yards.

70. The length of one side of a square room is 31 feet; how many square feet in the whole room? Ans. 961.

71. If the floor of a square room contain 36 square yards, how many feet does it measure on each side? Ans. 18 ft.

NOTE. This answer is obtained by finding the square root of the area 36 feet.

A *parallelogram*, or *oblong*, is a four sided figure, having its *opposite* sides equal and parallel.

To find the area of a parallelogram,

Multiply the length by the breadth.

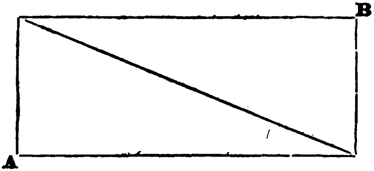
72. A garden in the form of a parallelogram is 96 feet long and 54 wide; how many square feet of ground are contained in it? Ans. 5184 sq. ft.

73. What is the area of a parallelogram 120 rods long and 60 wide? Ans. 7200 sq. rods.

74. If a board be 21 feet long, and 18 inches broad, how many square feet are contained in it? Ans. 31½ sq. ft.

A *triangle* is a figure bounded by three lines.

If a line be drawn from one corner of a parallelogram to its opposite, (as in the Fig. A B,) it will divide it into two equal parts of the same length and breadth as the parallelogram, but containing only half its surface. These two parts are triangles.—Now supposing the length of this parallelogram to be 6 feet, and its breadth 2, the area would be 12 feet. But the *triangle* will contain only half the surface, or 6 feet.



Hence to find the area of a triangle,

Multiply the length by half the breadth, or the breadth by half the length.

75. In a triangle 32 inches by 10, how many square inches? Ans. 160 sq. inches.

76. What is the area of a triangle whose base is 30 rods and the perpendicular 6 rods? Ans. 90 rods.

It has been shown that the length of one side of a cube raised to its *third power* will give the solid contents of the cube.

Hence to find the solid contents of a cube, when one side is given,

Multiply the given side into itself twice, or raise it to its third power.

77. The side of a cubic block is 12 inches; how many solid inches does the block contain? Ans. $12^3 = 1728$.

78. One side of a cube is 59 feet; what are its solid contents? Ans. 205379.

79. If a cube contains 614,125 cubic yards, what is the length of one side? Ans. 85 yards

NOTE. This answer is obtained by finding the cube root of 614125.

A *circle* is a figure contained by one line called the *circumference*, every part of which is equally distant from a point within called the *centre.*

The *diameter* of a circle, is a line drawn through the centre, dividing it into two equal parts.

It is found by calculation, that the *circumference* of a circle measures about $3\frac{1}{7}$ times as much as its diameter, or more accurately in decimals, 3,4159 times.

Hence to find the circumference of a circle when the diameter is known,

Multiply the diameter by $3\frac{1}{7}$.

To find the diameter when the circumference is known,

Divide the circumference by $3\frac{1}{7}$.

To find the *area* of a circle,

Multiply $\frac{1}{2}$ *the diameter into* $\frac{1}{2}$ *the circumference.*

80. If the diameter of a wheel is 4 feet, what is its circumference? Ans. $12\frac{4}{7}$ feet.

81. What is the circumference of a circle, whose diameter is 147 feet? Ans. 462 feet.

82. What is the diameter of a circle, whose circumference is 462 feet? Ans. 147 feet.

83. What is the area of a circle, whose diameter is 7 feet, and its circumference 22 feet? Ans. $38\frac{1}{2}$ sq. feet.

84. What is the area of a circle, whose circumference is 176 rods? Ans. 2464 rods.

The area of a *globe*, or *ball*, is 4 times as much as the area of a circle of the same diameter.

Hence, to find the area of a globe,

Multiply the whole circumference into the whole diameter.

85. What is the number of square miles on the surface of the earth, supposing its diameter 7911 miles?

Ans. $7911 \times 24863 = 196,612,083$.

To find the *solid contents* of a globe, or ball,

Multiply its area by $\frac{1}{6}$ *part of its diameter.*

86. How many solid inches in a ball 7 inches in diameter?

Ans. $179\frac{2}{3}$.

A *cylinder* is a round body, whose ends are circles, and which is of equal size from end to end.

To find the solid contents of a cylinder,

Multiply the area of one end by the length.

87. There is a cylinder 10 feet long, the area of whose ends is 3 square feet; how many solid feet does it contain? Ans. 30.

Solids which decrease gradually from the base till they come to a point, are called pyramids. The point at the top of a pyramid is called the *vertex*. A line drawn from the vertex perpendicular to the base, is called the *perpendicular height* of the pyramid.

To find the *solid contents* of a pyramid,

Multiply the area of the base by $\frac{1}{3}$ of the perpendicular height.

88. There is a pyramid whose height is 9 feet, and whose base is 4 feet square; what are its contents? Ans. 48 feet.

89. There is a pyramid, whose height is 27 feet, and whose base is 7 feet in diameter; what are its solid contents?

Ans. $346\frac{1}{3}$ feet.

FORMS OF NOTES, RECEIPTS, AND ORDERS.

When a man wishes to borrow money, after receiving it, he gives his promise to repay it, in such forms as those below.

NOTES.

No. 1.

Hartford, Jan. 1, 1832.

For value received, I promise to pay D. F. Robinson, or order, two hundred sixty-four dollars, twenty-five cents, on demand, with interest. JOHN SMITH.

No. 2.

New-York, Jan. 15, 1832.

For value received, I promise to pay William Dennis, or bearer, twenty dollars, sixteen cents, three months after date.

GEORGE ELLIS.

No. 3.

Philadelphia, July 6, 1831.

For value received, we, jointly, and severally, promise to pay to Henry Reddy, or order, one hundred dollars, thirteen cents, on demand, with interest. JAMES BARNES.

Attest. James Cook. WILLIAM HEDGE.

Remarks.

1. The sum lent, or borrowed, should be *written out in words*, instead of using figures.

2. When a note has the words "*or order*," or "*or bearer*," it is called *negotiable;* that is, it may be given or sold to another man, and he can collect it.

If the note be written, to pay him "*or order*," (see No. 1,) then D. F. Robinson can *endorse the note*, that is, write his name on the back of it, and then sell it to any one he chooses. Whoever buys the note, demands pay from the signer, John Smith.

3. If the note be written, "*or bearer*," (see note 2,) then whoever holds the note can collect it of the signer.

4. When no rate of interest is mentioned, it is to be understood at the legal rate in the state where the note is given.

5. All notes are payable on demand, unless some particular time is specified.

6. All notes draw interest after the time of promised payment has elapsed, even if there is no promise of interest in the note.

7. Notes that are to be paid *on demand*, draw interest after a demand is made.

8. If a man promises to pay in *certain other articles*, instead of money, after the time of promised payment has elapsed, the creditor can claim payment in *money*.

RECEIPTS.

Hartford, June 16, 1831.

Received of Mr. Julius Peck, twelve dollars, in full of all accounts. JOHN OSGOOD.

Receipt for money on a note.

Hartford, June 18, 1831.

Received of John Goodman, (by the hand of William Smith,) twenty dollars, sixteen cents, which is endorsed on his note of July 6, 1829. JOHN REED.

Receipt for money on account.

Hartford, April 6, 1831.

Received of Albert Jones, forty dollars, on account.

PETER TRUSTY.

Receipt for money for another person.

Hartford, June 1st, 1831.

Received of A. B. one hundred and six dollars, for I. C.

SAMUEL WILSON.

Receipt for interest due on a note.

Hartford, Aug. 1, 1832.

Received of W. B. thirty dollars in full of one year's interest of $ 500, due to me on the —— day of —— last, on note from the said W. B. WILLIAM GRAY.

Receipt for money paid before it is due.

Newport, June 1, 1829.

Received of A. F. sixty dollars advanced, in full for one year's rent of my house, leased to said A. F. ending the first day of September next, 1829. JOHN GRAVES.

NOTE.—If a receipt is given *in full of all accounts*, it cuts off only the claims of *accounts*. But "*in full of all demands*" cuts off all claims of every kind.

ORDERS.

New York, June 9, 1830.

Mr. John Ayers. For value received, pay to N. S. or order, fifty dollars, and place the same to my account.

SOLOMON GREEN.

New York, July 9, 1831.

Mr. William Redfield,—Please to deliver Mr. L. D. such goods as he may call for, not exceeding the sum of one hundred dollars, and place the same to the account of your humble servant. STEPHEN BIRCH.

BOOK-KEEPING.

When accounts are disputed in business, it is necessary to produce the book in the courts where the charge was *first made*, and produce legal evidence of their correctness. The kind of evidence demanded differs in different states.

The following is an easy and simple method of keeping accounts for farmers, mechanics, &c.

Take a book, ruled as below. Enter the name of the person with whom you are to open an account, at the top of the *left* hand page, as *Dr.*, and at the top of the *right* as *Cr.* Thus;

Dr. John Good.

1827.		$	cts.
Jan. 5.	To 5 cords of wood, at $1,75,	8	75
May 16.	To one day's work, self and oxen,	1	50
July 23.	To 4 bushels of rye, at 75 cts. delivered by your order to C. D.	3	00
		13	25

John Good. Cr.

1827.		$	cts.
April 8.	By one Plough,	9	25
May 10.	By repairing Cart Wheels,	1	50
Sept. 12.	By Cash to balance,	2	50
		13	25

It is very important that females should know how to keep accounts properly. The following is a simple method.

Let the book be ruled as below.

Put your name as *Dr*. on the left hand, and as *Cr*. on the right hand page. Consider yourself as *Dr*. for every thing *received*, and as *Cr*. for every thing *paid out*.

Dr. **Mary Trusty.**

1831.		$	cts.
May 1.	To cash rec'd from my father,	15	00
16.	To do. do.	3	50
29.	To do. do.	3	25
June 1.	To do, do.	10	00
		31	75

Mary Trusty. **Cr.**

1831.		$	cts.
May 6.	By cash paid for 16 lbs. of butter at 20 cents per lb.	3	20
29.	By do. for 200 bushels of coal at 6 cents per bushel,	12	00
	By do. for 1 barrel of soap,	4	25
	By cash on hand,	12	30
		31	75

This book should be returned to the Library on or before the last date stamped below.

A fine of five cents a day is incurred by retaining it beyond the specified time.

Please return promptly.

...tely published ... belonging to the series of ... By J. L. Comstock, M. ...
...es for Youth."

...TH'S BOOK OF NATURAL PHILOSOPHY.

...design of this little Book is to afford a facility to the ...uction of Natural Philosophy into common schools, ... has not heretofore existed in our country. ... distinct sciences, the elements of which are here ex...ed, are Mechanics, Hydrostatics, Hydraulics, Pneumat... Acoustics, Optics and Astronomy; each being illustrated ...umerous diagrams, and it is hoped made as plain, and ...ly understood, as the nature of the cases will allow. ...he want of this kind of knowledge is deeply regretted ... every reflecting, uneducated person in himself; for such ...rsons every day observe facts, which though mysterious ... themselves, they know are readily explained by those who ...ave even but a slight knowledge of Natural Philosophy.

YOUTH'S BOOK OF ASTRONOMY.

It is true that several small books on this science, "designed ... for beginners," are already before the public. But the ... Author of this, has seen none which he thought brought the ... down to the capacity of youth, and explained them ... sible manner. ... with the Moon, which is obviously ... ial object, which youth ...